MODAL ANALYSIS OF ROTOR STRUCTURE DYNAMICS

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The newly developed modal analysis methods applied to rotor dynamics problems allow reduction of calculation and time expenses. The algorithms take into consideration changes of inertia rotor parameters, stiffness performance, gyroscopic and damping forces. The proposed algorithm includes consideration of rotor system axial loads

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Mathematical models applied to rotor dynamics simulation must comply with specific requirements. One of them is the efficiency capability in solution of various rotor dynamics problems with large numbers of degrees of freedom. The models must accurately describe elastic and inertia system parameters, links between elements, different types of oscillations, operating conditions and loads.

In the rotor systems dynamics analysis are widely used calculation models based on such methods as finite elements method, initial parameters method, modal analysis and synthesis [1].

The modal methods allow remarkable reduction of calculation expenses, so they are especially important in the rotor dynamics problems. These methods present the linear system dynamic behavior through the natural modes decomposition. Application of a complete set of natural modes to the modal method is equivalent to solution of the discrete dynamic system motion equations, and the solution is accurate. A limited set of lower frequencies natural modes also may provide needed solution accuracy. A compromise between the solution accuracy and the labor expenses is one of the modal methods advantages.

Modal motion equations. In general terms the discontinuous system motion equation may be written as the following:

$$M\ddot{q} + Kq = Q(t)\,, \qquad (1)$$

where M - inertia matrix, K – stiffness matrix, q – column of generalized motions of stations where are located the system inertia elements, Q – column of generalized external forces applied to the system stations.

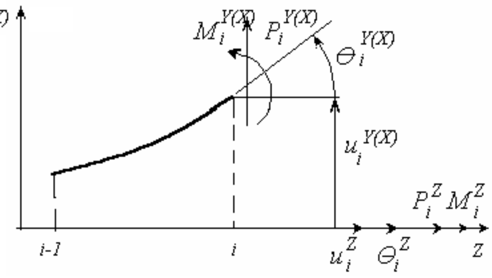
Columns q and Q of the equation (1) have the following structures:

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \dots \\ q_n \end{bmatrix};\; q_i = \begin{bmatrix} u_i^X \\ \theta_i^X \\ u_i^Y \\ \theta_i^Y \\ u_i^Z \\ \vartheta_i^Z \end{bmatrix};$$

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \\ \dots \\ Q_n \end{bmatrix};\; Q_i = \begin{bmatrix} P_i^X \\ M_i^X \\ P_i^Y \\ M_i^Y \\ P_i^Z \\ M_i^Z \end{bmatrix}, \qquad (2)$$

where q_i - column of generalized motions in i stations, Q_i - column of generalized external forces in i stations, n – number of the system stations.

Columns q_i and Q_i consist of projection components of linear u and angular θ motions and the related components of external force P and external moment M. Positive axis directions are shown in picture 1.



Pic. 1

If the system natural frequencies and modes satisfy equation (1) at $Q = 0$, an approximate equation (1) solution may be presented as the following natural modes decomposition:

$$q = \sum_{i=1}^{r} \bar{q}(i)e(i) = \bar{q}e\;, \qquad (3)$$

where $\bar{q}$ =[$\bar{q}$ (*1*), $\bar{q}$ (*2*)… $\bar{q}$ (*r*)] – matrix of the system oscillation natural modes, or modal matrix, $\bar{q}$ *(i)* – column of motions that describes the *i*-th mode, *e* – modal coordinates column, *r* – number of modal coordinates.

Separation of proper lines from matrix $\bar{q}$ allows expansion of linear and angular motions projections on modal coordinates in any station of the system:

$$u_i^X = \bar{u}_i^X e; \qquad u_i^Y = \bar{u}_i^Y e;$$

$$u_i^Z = \bar{u}_i^Z e; \qquad (4)$$

$$\theta_i^X = \overline{\theta}_i^X e; \quad \theta_i^Y = \overline{\theta}_i^Y e;$$
$$\theta_i^Z = \overline{\theta}_i^Z e, \quad i = 1,2,\dots,n.$$

The natural modes orthogonality leads to the following conditions:

$$\overline{q}^T M \overline{q} = \overline{M} = diag(\overline{m}(i));$$
$$\overline{q}^T K \overline{q} = \overline{K} = diag(\overline{m}(i) p^2(i)), \qquad (5)$$
$$i = 1...m,$$

where $\overline{m}(i),\ p(i)$ - reduced mass and natural frequency of the $\overline{q}$ *(i).* mode. Here the index ^{T} means the transpose operation.

Substitution of (2) into (1) and multiplying the result by $\overline{q}^T$, results in the system motion equations in modal coordinates:

$$\overline{M}\ddot{e} + \overline{K}e = \overline{Q}(t), \qquad (6)$$

where $\overline{M}$ and $\overline{K}$ - modal, or adjusted to modal coordinates, inertia and stiffness matrixes defined by relation (5), $\overline{Q}(t)$ - modal column of generalized external forces calculated as the following:

$$\overline{Q}(t) = \overline{q}^T Q(t). \qquad (7)$$

Increase of modal coordinates *r* in (5) leads to a more accurate modal equation (6) but the equation dimensionality and the solution laboriousness also grow. Change of the modal co-ordinates number allows a compromise between the solution accuracy and the calculation labor.

The modal extension method allows easier analysis of the problems concerned to elimination of dangerous resonances by changes of elements inertia and stiffness performances. Also it is no so difficult to take into account influences of gyroscopic moments or static loads including longitudal tension and compression forces, or of the system non-linear elements like journal bearings, dampers and seals. All these problems may be solved on the same model basis by addition of proper modal matrixes. The problem dimensionality is determined by the modal coordinates number in the modal motion equation, so it may be kept constant.

Below are disclosed algorithms of additional modal matrixes used for some typical problem.

Modal matrix for changes of inertia parameters. The inertia elements of the discontinuous system model are concentrated in the calculation stations and are specified by inertia matrixes that relate inertia forces and moments with the stations accelerations.

Changes of inertia forces and moments caused by changes of the inertia parameters of the *i* station element may be presented by additional forces and moments calculated by the following formulae:

$$\begin{bmatrix} \Delta P^X(i) \\ \Delta M^X(i) \\ \Delta P^Y(i) \\ \Delta M^Y(i) \\ \Delta P^Z(i) \\ \Delta M^Z(i) \end{bmatrix} = \Delta M(i) \begin{bmatrix} \ddot{u}^X(i) \\ \ddot{\theta}^X(i) \\ \ddot{u}^Y(i) \\ \ddot{\theta}^Y(i) \\ \ddot{u}^Z(i) \\ \ddot{\theta}^Z(i) \end{bmatrix} = \left(\Delta M(i) \begin{bmatrix} \overline{u}^X(i) \\ \overline{\theta}^X(i) \\ \overline{u}^Y(i) \\ \overline{\theta}^Y(i) \\ \overline{u}^Z(i) \\ \overline{\theta}^Z(i) \end{bmatrix} \right) \ddot{e} \qquad (8)$$

where $\Delta M(i)$ - change of the *i* element inertia matrix. A more complete expression for the $\Delta M(i)$ matrix is expressed as

$$\Delta M(i) = \Delta \begin{bmatrix} m & S^Z & 0 & 0 & 0 & -S^Y \\ S^Z & J^Y & 0 & J^{YZ} & -S^X & J^{YZ} \\ 0 & 0 & m & S^Z & 0 & S^X \\ 0 & J^{XY} & S^Z & J^X & -S^Y & J^{XZ} \\ 0 & -S^X & 0 & -S^Y & m & 0 \\ -S^Y & J^{ZY} & S^X & J^{ZX} & 0 & J^Z \end{bmatrix} (i), \qquad (9)$$

where m – mass; S^X, S^Y, S^Z - static moments in planes *YZ, ZX, XY*; J^X, J^Y, J^Z - inertia moments in axes *X, Y, Z*; $J^{YZ} = J^{ZY}$, $J^{ZX} = J^{XZ}$, $J^{XY} = J^{YX}$ - centrifugal inertia moments.

Calculation by formulae (7) of the additional forces modal column related to changes of the elements inertia parameters gives:

$$\Delta\overline{Q} = -\left(\sum_i \begin{bmatrix} \overline{u}^X(i) \\ \overline{\theta}^X(i) \\ \overline{u}^Y(i) \\ \overline{\theta}^Y(i) \\ \overline{u}^Z(i) \\ \overline{\theta}^Z(i) \end{bmatrix}^T \Delta M(i) \begin{bmatrix} \overline{u}^X(i) \\ \overline{\theta}^X(i) \\ \overline{u}^Y(i) \\ \overline{\theta}^Y(i) \\ \overline{u}^Z(i) \\ \overline{\theta}^Z(i) \end{bmatrix} \right) \ddot{e}. \qquad (10)$$

Equation (10) shows that the additional generalized modal forces described by the column $\Delta\overline{Q}$ are proportional to the modal accelerations $\ddot{e}$ and thus may be considered as additional modal generalized inertia forces created by the additional modal inertia matrix $\Delta\overline{M}$ calculated by formulae

$$\Delta\overline{M} = \sum \Delta\overline{M}(i), \qquad (11)$$

where

$$\Delta\overline{M}(i) = \begin{bmatrix} \overline{u}^X(i) \\ \overline{\theta}^X(i) \\ \overline{u}^Y(i) \\ \overline{\theta}^Y(i) \\ \overline{u}^Z(i) \\ \overline{\theta}^Z(i) \end{bmatrix}^T \Delta M(i) \begin{bmatrix} \overline{u}^X(i) \\ \overline{\theta}^X(i) \\ \overline{u}^Y(i) \\ \overline{\theta}^Y(i) \\ \overline{u}^Z(i) \\ \overline{\theta}^Z(i) \end{bmatrix}. \qquad (12)$$

Matrix $\Delta\overline{M}(i)$ reflects contribution of *i* inertia element into the total additional modal inertia matrix $\Delta\overline{M}$.

The system motion equation in modal coordinates with changes of its element inertia parameters may be written as

$$\left(\overline{M}+\Delta\overline{M}\right)\ddot{e}+\overline{K}e=\overline{Q}(t). \quad (13)$$

Modal matrix for changes of stiffness parameters. Changes of elements stiffness parameters may be presented as application of additional non-inertial links that block angular and linear relative motions of the system stations.

The additional link of the element *i* may be resented as a scheme where one of the element boundary stations $n_1(i)$ is assumed absolutely unmovable, or embedded, and the other boundary station $n_2(i)$ is free.

In the free boundary station the stiffness matrix $\Delta K(i)$ sets a relation between the station motions, or the link displacements, and the loads that cause these displacements.

$$\begin{bmatrix}\Delta P^X(i)\\ \Delta M^X(i)\\ \Delta P^Y(i)\\ \Delta M^Y(i)\\ \Delta P^Z(i)\\ \Delta M^Z(i)\end{bmatrix}=\Delta K(i)\begin{bmatrix}\Delta u^X(i)\\ \Delta\theta^X(i)\\ \Delta u^Y(i)\\ \Delta\theta^Y(i)\\ \Delta u^Z(i)\\ \Delta\theta^Z(i)\end{bmatrix}. \quad (14)$$

The link displacement column may be expressed by motions of its boundary stations $n_1(i)$ and $n_2(i)$ in modal coordinates.

$$\begin{bmatrix}\Delta u^X(i)\\ \Delta\theta^X(i)\\ \Delta u^Y(i)\\ \Delta\theta^Y(i)\\ \Delta u^Z(i)\\ \Delta\theta^Z(i)\end{bmatrix}=\begin{bmatrix}\Delta\overline{u}^X(i)\\ \Delta\overline{\theta}^X(i)\\ \Delta\overline{u}^Y(i)\\ \Delta\overline{\theta}^Y(i)\\ \Delta\overline{u}^Z(i)\\ \Delta\overline{\theta}^Z(i)\end{bmatrix}e=\begin{bmatrix}\overline{u}^X_{n_2(i)}-\overline{u}^X_{n_1(i)}-L_i\overline{\theta}^X_{n_1(i)}\\ \overline{\theta}^X_{n_2(i)}-\overline{\theta}^X_{n_1(i)}\\ \overline{u}^Y_{n_2(i)}-\overline{u}^Y_{n_1(i)}-L_i\overline{\theta}^Y_{n_1(i)}\\ \overline{\theta}^Y_{n_2(i)}-\overline{\theta}^Y_{n_1(i)}\\ \overline{u}^Z_{n_2(i)}-\overline{u}^Z_{n_1(i)}\\ \overline{\theta}^Z_{n_2(i)}-\overline{\theta}^Z_{n_1(i)}\end{bmatrix}e, \quad (15)$$

where L_i - distance between the stations $n_1(i)$ and $n_2(i)$ along Z axis.

The additional forces modal column due action of additional links is calculated by formulae (7):

$$\Delta\overline{Q}=-\left(\sum_i\begin{bmatrix}\Delta\overline{u}^X(i)\\ \Delta\overline{\theta}^X(i)\\ \Delta\overline{u}^Y(i)\\ \Delta\overline{\theta}^Y(i)\\ \Delta\overline{u}^Z(i)\\ \Delta\overline{\theta}^Z(i)\end{bmatrix}^T\Delta K(i)\begin{bmatrix}\Delta\overline{u}^X(i)\\ \Delta\overline{\theta}^X(i)\\ \Delta\overline{u}^Y(i)\\ \Delta\overline{\theta}^Y(i)\\ \Delta\overline{u}^Z(i)\\ \Delta\overline{\theta}^Z(i)\end{bmatrix}\right)e. \quad (16)$$

Equation (16) shows that the additional modal generalized forces described by the column $\Delta\overline{Q}$ are proportional to the modal displacements *e* and thus may be considered as reactions of the additional modal stiffness $\Delta\overline{K}$ calculated by the following formulae:

$$\Delta\overline{K}=\sum_i\Delta\overline{K}(i), \quad (17)$$

where

$$\Delta\overline{K}(i)=\begin{bmatrix}\Delta\overline{u}^X(i)\\ \Delta\overline{\theta}^X(i)\\ \Delta\overline{u}^Y(i)\\ \Delta\overline{\theta}^Y(i)\\ \Delta\overline{u}^Z(i)\\ \Delta\overline{\theta}^Z(i)\end{bmatrix}^T\Delta K(i)\begin{bmatrix}\Delta\overline{u}^X(i)\\ \Delta\overline{\theta}^X(i)\\ \Delta\overline{u}^Y(i)\\ \Delta\overline{\theta}^Y(i)\\ \Delta\overline{u}^Z(i)\\ \Delta\overline{\theta}^Z(i)\end{bmatrix}. \quad (18)$$

Matrix $\Delta\vec{K}(i)$ describes contribution of the element *i* stiffness changes into the total additional modal stiffness $\Delta\overline{K}$.

The system motion equation in modal coordinates with changes of the elements stiffness parameters may be written as

$$\overline{M}\ddot{e}+\left(\overline{K}+\Delta\overline{K}\right)e=\overline{Q}(t). \quad (19)$$

Modal matrix with damping and gyroscopic effect. The system may be assumed as consisting of n_S sub-systems located along axis Z connected to each other with n_L links. The system damping is provided by dampers located in parallel with the elastic sub-systems links. Each sub-system has its own rotation speed ω. Additional damping forces caused by actions of the applied dampers and gyroscopic moments caused by the subsystems rotation are calculated by (7). In the system modal motion equation appears an additional modal matrix $\overline{C}$ for damping and gyroscopic effects. The modal motion equation then becomes:

$$\overline{M}\ddot{e}+\overline{C}\dot{e}+\overline{K}e=\overline{Q}(t) \ . \quad (20)$$

Matrix $\overline{C}$ may be presented as a sum of two matrixes:

$$\overline{C}=\overline{C}d+\overline{C}g=\sum_{i=1}^{n_L}\overline{C}d(i)+\sum_{i=1}^{n_S}[\omega(i)\overline{C}g_1(i)] \ , \quad (21)$$

where $\overline{C}d$ - total modal damping matrix; $\overline{C}g$ - total modal gyro matrix; $\overline{C}d(i)$ - modal damping matrix of the damper *i*; $\overline{C}g_1(i)$ - modal gyro matrix of the syb-system *i* calculated for the rotation speed unit, $\omega(i)$ - rotation speed for the sub-system *i.*

Matrix $\overline{C}d(i)$ is calculated by formulae:

$$\overline{C}d(i)=\begin{bmatrix}\Delta\overline{u}^{X}(i)\\ \Delta\overline{\theta}^{X}(i)\\ \Delta\overline{u}^{Y}(i)\\ \Delta\overline{\theta}^{Y}(i)\\ \Delta\overline{u}^{Z}(i)\\ \Delta\overline{\theta}^{Z}(i)\end{bmatrix}^{T} C(i)\begin{bmatrix}\Delta\overline{u}^{X}(i)\\ \Delta\overline{\theta}^{X}(i)\\ \Delta\overline{u}^{Y}(i)\\ \Delta\overline{\theta}^{Y}(i)\\ \Delta\overline{u}^{Z}(i)\\ \Delta\overline{\theta}^{Z}(i)\end{bmatrix}. \qquad (22)$$

Formulae (22) is obtained by the scheme used for the similar formulae (18) but the stiffness matrix $\Delta K(i)$ is replaced by the damping matrix *C(i)*, the modal displacement *e* is replaced by the modal velocity $\dot{e}$. The damper displacement is calculated through motions of its boundary stations $n_1(i)$ and $n_2(i)$.

When a damper operates only with linear deflections directed along the axes X and Y, matrix $\overline{C}d(i)$ may be written as

$$\overline{C}d(i)=\begin{bmatrix}\Delta\overline{u}^{X}(i)\\ \Delta\overline{u}^{Y}(i)\end{bmatrix}^{T}\begin{bmatrix}C^{XX}(i) & C^{XY}(i)\\ C^{YX}(i) & C^{YY}(i)\end{bmatrix}\begin{bmatrix}\Delta\overline{u}^{X}(i)\\ \Delta\overline{u}^{Y}(i)\end{bmatrix}, \qquad (23)$$

where

$$\Delta\overline{u}^{X}(i)=\overline{u}^{X}_{n_2(i)}-\overline{u}^{X}_{n_1(i)}-L_i\overline{\theta}^{X}_{n_1(i)}; \qquad \Delta\overline{u}^{Y}(i)=\overline{u}^{Y}_{n_2(i)}-\overline{u}^{Y}_{n_1(i)}-L_i\overline{\theta}^{Y}_{n_1(i)}.$$

Matrix $\overline{C}g_1(i)$ is calculated by

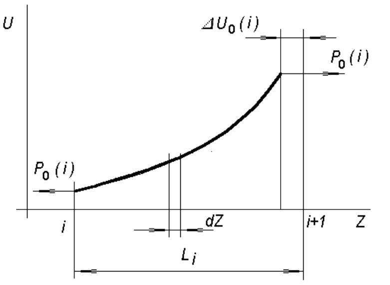
$$\overline{C}g_1(i)=\sum_{k=n_1(i)}^{n_K(i)}\left(J_p(k)\left[(\overline{\theta}^{X}_k)^T\,\overline{\theta}^{Y}_k-(\overline{\theta}^{Y}_k)^T\,\overline{\theta}^{X}_k\right]\right), \qquad (24)$$

where $n_1(i)$, $n_K(i)$ – first and last numbers of the sub-system <u>i</u> stations<u>;</u> $J_p(k)$ - polar inertia moment of the disc located in station *k;* $\overline{\theta}^{X}_k$, $\overline{\theta}^{Y}_k$ - decomposition of the station *k* angular motions in planes *XZ* and *YZ.*

In formulae (23) axes *X, Y, Z* form the right coordinate system, the positive rotation direction corresponds to the shortest *X* motion to the coincidence with the *Y* axis.

Modal matrix for stationary axial loads. Stationary axial loads applied to sub-systems change their elastic resistance under transversal deflections. Tension loads increase the leg stiffness, compressing loads reduce it. As the result the natural frequency spectrum, resonance modes and the system vibration response also change. Large compression loads may cause the system buckling when the transversal deflections unlimitedly increase, even without any transversal loads.

Picture 2 shows the system element *i* located between stations *i* and *i+1*.



Pic. 2

The beam has linear and angular displacements and deflections in the transversal direction in plane *XY* and is loaded with an axial, in *Z* direction tension force $P_0(i)$. The displaced and deflected beam projection to the *Z* axis is smaller than its length, so the distance between the beam projection end points shortens for $\Delta U_0(i)$. The applied axial tension force $P_0(i)$ acts against the axial displacement $\Delta U_0(i)$, thus it increases the system stiffness. At negative, or compression values of $P_0(i)$ its effect is the system stiffness reduction.

Changes of the system stiffness under the axial forces applied to the elements may be expressed with an additional stiffness matrix $\Delta\overline{K}_0$ in the modal equation (6) that reaches the following form:

$$\overline{M}\ddot{e}+(\overline{K}+\Delta\overline{K}_0)e=\overline{Q}(t), \qquad (25)$$

Here the additional modal matrix $\Delta\overline{K}_0$ is calculated by the formulae

$$\Delta\overline{K}_0=\sum_i\left[P_0(i)\Delta\overline{K}_{01}(i)\right], \qquad (26)$$

where $P_0(i)$ – axial tension, or at negative values compression force applied to the system element *I*; $\Delta\vec{K}_{01}(i)$ - modal stiffness matrix of the element *i* loaded with a unit tension force.

The sum is calculated throughout all elements loaded with stationary axial forces.

The algorithm for the matrix $\Delta\vec{K}_{01}(i)$ calculation may be taken from the following subsequent equations:

$$\Delta\overline{K}_0=-\frac{d}{de}\overline{Q}_0=-\sum_i\frac{d}{de}\overline{Q}_0(i)=-\sum_i\frac{d}{de}\left[\Delta\overline{U}_0(i)P_0(i)\right]=-\sum_i\left[P_0(i)\frac{d^2}{de^2}\Delta U_0(i)\right]. \qquad (27)$$

A comparison of (26) and (27) results in

$$\Delta\overline{K}_{01}(i)=-\frac{d^2}{de^2}\Delta U_0(i). \qquad (28)$$

So the matrix $\Delta\vec{K}_{01}(i)$ may be obtained through axial displacement $\Delta U_0(i)$ expressed in modal coordinates.

With shear deflection neglected the following are the motion relations (picture 3):

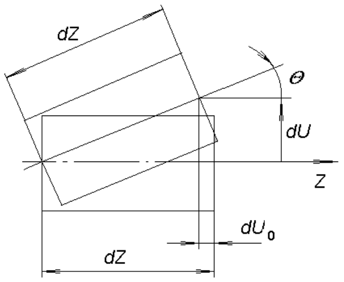
$$dUo = dz\cos\theta - dz\,;$$
$$du = dz\sin\theta\,,$$

At small θ values it gives

$$\frac{dUo}{dz} = -\frac{1}{2}\theta^2 = -\frac{1}{2}\left[\left(\theta^X\right)^2 + \left(\theta^Y\right)^2\right];$$

(29)

$$\theta^X = \frac{du^X}{dz}\,; \qquad \theta^Y = \frac{du^Y}{dz}\,. \qquad (30)$$



Pic. 3

Transversal displacements of the element *i* elastic curve $u^X(z)$ and $u^Y(z)$ may be presented as a cubic polynomial:

$$u(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3\,, \qquad (31)$$

with the coefficients a_0, a_1, a_2, a_3 determined by the boundary conditions:

$$u(0) = u_i\,; \qquad \theta(0) = \theta_i\,; \qquad u(L) = u_{i+1}\,; \qquad \theta(L) = \theta_{i+1}\,,$$

(32)

where $L = L_i$ - element length.

Equations (30) and (31) allow a matrix equation form for the boundary conditions:

$$G\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} u_i \\ \theta_i \\ u_{i+1} \\ \theta_{i+1} \end{bmatrix}, \text{ где } G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & L & L^2 & L^3 \\ 0 & 1 & 2L & 3L^2 \end{bmatrix}. \qquad (33)$$

Coefficient a_0, a_1, a_2, a_3 obtained from (33) and substituted into (31) result in

$$u(z) = [1 \;\; z \;\; z^2 \;\; z^3]BS(i)\,, \qquad (34)$$

where $S(i) = \begin{bmatrix} u_i \\ \theta_i \\ u_{i+1} \\ \theta_{i+1} \end{bmatrix}; \qquad B = G^{-1}\,;$

index $^{-1}$ means the matrix inversion.

Derivative of (31) on *z* gives

$$\theta(z) = [0 \;\; 1 \;\; 2z \;\; 3z^2]BS(i)\,. \qquad (35)$$

Equations (31) to (35) for the transversal deflections in directions *X* and *Y* are similar, their upper indexes may be *X* or *Y*. Replacement of *S(i)* by $S^X(i) = \bar{S}^X(i)e$ or $S^Y(i) = \bar{S}^Y(i)e$ allows expression of (35) through modal coordinates $\theta^X(z)$ or $\theta^Y(z)$:

$$\theta^X(z) = \bar{\theta}^X(z)e\,; \qquad (36)$$
$$\theta^Y(z) = \bar{\theta}^Y(z)e\,, \qquad (37)$$

where $\bar{\theta}^X(z) = [0 \;\; 1 \;\; 2z \;\; 3z^2]B\bar{S}^X(i)\,;$ (38)

$$\bar{\theta}^Y(z) = [0 \;\; 1 \;\; 2z \;\; 3z^2]B\bar{S}^Y(i)\,. \qquad (39)$$

Substitution of $\theta^X(z)$ and $\theta^Y(z)$ into (29) and integration along the element length result in the displacement $\Delta U_0(i)$ expressed in modal coordinates:

$$\Delta U_0(i) = -\frac{1}{2}e\left(\int_0^L \left[\left(\bar{\theta}^X\right)^T\bar{\theta}^X + \left(\bar{\theta}^Y\right)^T\bar{\theta}^Y\right]dz\right)e$$

(40)

Equation (40) may be substituted into (28):

$$\Delta\bar{K}o1(i) = \int_0^L \left[\left(\bar{\theta}^X\right)^T\bar{\theta}^X + \left(\bar{\theta}^Y\right)^T\bar{\theta}^Y\right]dz\,. \qquad (41)$$

Finally the matrix $\Delta\bar{K}_0(i)$: is obtained by replacement of $\bar{\theta}^X$ and $\bar{\theta}^Y$ by (37) and (38):

$$\Delta\bar{K}_0(i) = \left(\bar{S}^X(i)\right)^T J(i)\bar{S}^X(i) + \left(\bar{S}^Y(i)\right)^T J(i)\bar{S}^Y(i)\,,$$

(42)

where $\bar{S}^X(i)$, $\bar{S}^Y(i)$ - modal matrixes to describe transversal displacements of the element *i* end stations:

$$\bar{S}^X(i) = \begin{bmatrix} \bar{u}^X{}_i \\ \bar{\theta}^X{}_i \\ \bar{u}^X{}_{i+1} \\ \bar{\theta}^X{}_{i+1} \end{bmatrix}; \bar{S}^Y(i) = \begin{bmatrix} \bar{u}^Y{}_i \\ \bar{\theta}^Y{}_i \\ \bar{u}^Y{}_{i+1} \\ \bar{\theta}^Y{}_{i+1} \end{bmatrix}; \qquad (43)$$

J(i) – matrix depending upon the element *i* length $L = L_i$ that is calculated as

$$J(i) = \int_0^L \left([0 \;\; 1 \;\; 2z \;\; 3z^2]B\right)^T\left([0 \;\; 1 \;\; 2z \;\; 3z^2]B\right)dz =$$

(44)

$$= \frac{1}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}.$$

The final motion modal equation with variable inertia and stiffness element parameters, damping, gyroscopes, and axial tension or compression loads is the following:

$$\left(\bar{M} + \Delta\bar{M}\right)\ddot{e} + \bar{C}\dot{e} + \left(\bar{K} + \Delta\bar{K} + \Delta\bar{K}o\right)e = \bar{Q}(t)$$

(45)

Conclusion

The considered modal analysis algorithms cover all main problems of linear rotor system analysis. Their main advantage against the direct calculations is a drastic reduction of calculation expenses at price of minor accuracy reduction. This makes a base for their application to the non-linear and unsteady rotor dynamics problems when the motion equations are directly integrated. The algorithms are used in the Dynamics R4 computer code proposed for solutions of linear, non-linear and unsteady rotor dynamics problems [2].

References

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