ROTOR-BEARING DYNAMICS TECHNOLOGY

DESIGN GUIDE

DAMPER SUPPORTS

1996

FOREWORD

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CONTENTS

1. INTRODUCTION

Using of Damper Supports (DS) is the most effective way to decrease vibration level and dynamical stresses in engine parts. A rather difficult problem of DS design can be separated into three main stages as the following:

1. Decision to use a DS: DS locating, definition of desired stiffness and damping.
2. DS design: choice of the DS type, calculation of its main dimensions, detailed design.
3. DS manufacturing and testing, including engine tests.

Purpose of this Guide is to disclose the milestone points, DS layouts and design, to disclose current possibilities in DS design.

2. DAMPER SUPPORTS TYPES

Main functions of various DSs are:

- Reduction of a support stiffness and thus changing of the engine dynamic system. The natural frequencies are reduced, resonances are removed from the operating ranges.
- Absorbing of the vibrating system energy by transferring it into heat. This does not allow large vibration amplitudes, and thus large loads and stresses in the engine parts.

Main requirements to aviation engines DSs are the following:

- The support shall have a particular stiffness, which is determined by the engine Dynamics analysis. The analysis result is removing of the rotor resonances from the engine operating range.

The support shall have a particular damping to avoid large vibrations at various operation conditions, including extreme conditions.

The support shall permit sufficient radial displacements. The needed radial displacement is determined by a formulae:

$$\delta_0 = \frac{mn}{k} g + k_d e \,,$$

Here

k - support stiffness coefficient; m - rotor mass; n - vertical overload coefficient due to an airframe evolutions (for example, cargo planes have n = 2 to 2.5); g - gravitational acceleration; k_d - dynamic amplifying coefficient for rotor vibrations in the support (k_d = 4 to 6); e - eccentricity of the rotor unbalance, which is taken equal to maximal tolerance, including operating unbalancing. It is defined by permittable unbalance $(me)_d$ on the particular support:

$$e = \frac{(me)_d}{m} \,.$$

- To avoid great radial displacements and flexible elements overstressing the displacements are limited by special limitators which come to operation only at extreme non-design conditions.
- To avoid the rotor and stator non-axiality the datum surfaces of flexible elements are preliminary displaced as to equalize statical displacements due to the rotor weight.
- The dapmer design shall have specific features permitting to control its performances and to optimize them at the engine development.
- Stability of supports dynamical performances requires high accuracy of manufacturing of flexible elements, fitting surfaces, throttle holes etc. For this purpose are used high classes of accuracy, selective assemblying and other specific technologies.

2.1 Hydrodynamic damper supports.

Hydrodynamic damper supports can be located in compressor and/or turbine areas (Fig. 1).

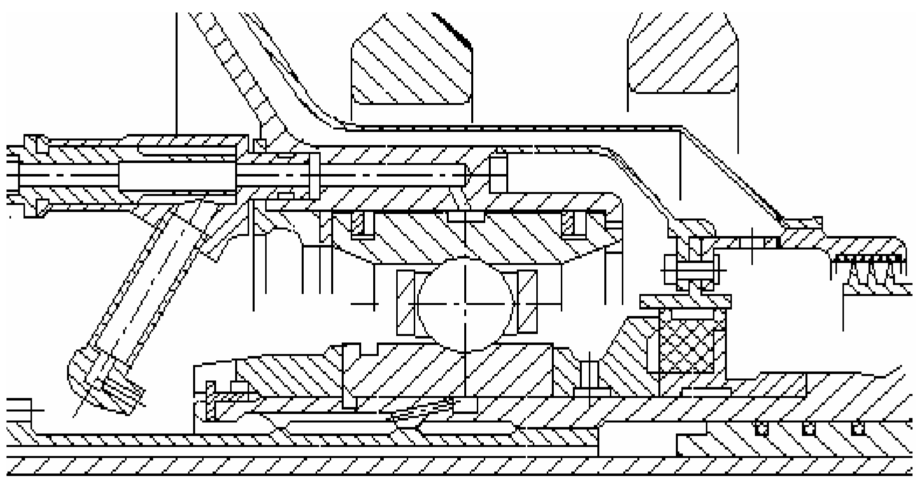

Fig.1 Compressor support with Hydrodynamic damper

Damping function is performed by a 0.1 to 0.3 mm thick oil film between the bearing outer ring and the casing. The oil film thickness determines the support radial displacement. The oil is high pressure supplied into the spacing between the bearing outer ring and the casing from an annual groove. The damper is sealed by two piston rings which reduce oil consumption and permit the rotor precession within the spacing.

At engine operating in the film there is produced a hydraulic force, so the oil film can transmit radial loads to the casing. On the other side the hydraulic force resists to the rotor precession in the support, i.e. a resisting or damping force is produced. A specific feature of such supports is that they are not to transmit axial loads when installed with roller-bearings or transmit small axial loads when installed with thrust ball-bearings in terms of not to meet self-stopping effect.

The oil film in the damper is highly loaded which is followed by significant production of heat. It has an influence on the oil viscosity at different engine operating conditions. So to permit the support stability the sealing rings are to allow nesessary oil massflow.

The oil massflow through the support depends on supplying pressure, oil temperature and main damper dimensions and fittings. Usually the massflow in different engines is within 10 to 100 kg/hr.

Dampers without seals need 2 to 3 times greater oil consumption and are seldom used in aviation engines but are widely used in marine application engines.

The temperature state and oil consumption are investigated experimentally.

Support dynamic performances (stiffness *k* and damping *c*) are nonlinear and depend on rotor location in the damper clearance (Fig.2).

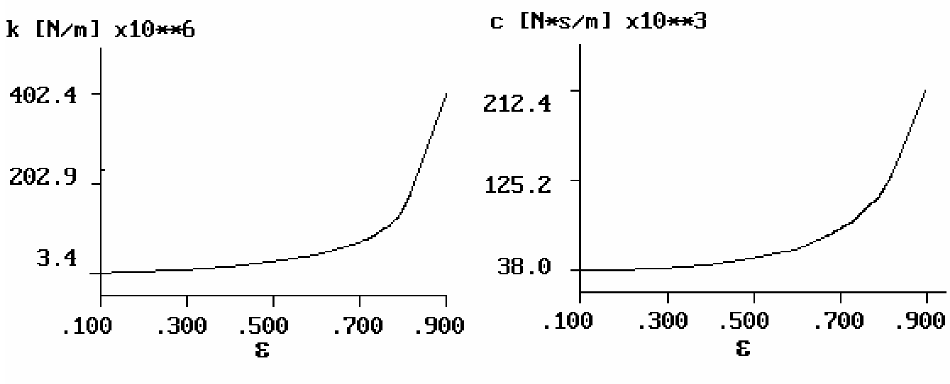

Fig. 2 Example of damper dynamic characteristics

$\varepsilon = \dfrac{e}{\delta}$ - eccentricity ratio; e - eccentricity between journal centre and housing centre; δ- radial clearance (housing radius - journal radius).

These supports advantages are small size, reliability, high damping and bearing ability, availability of mathematical models. The main shortcoming is nonlinear dynamic performances - the support operates from some speed of rotor rotation, without rotation there is no rotor centering, so the blades tip clearance is to be greater than the damper clearance.

Are known hydrodynamic supports with two or more oil films separated by free rings. Such supports have good bearing ability and damping and can be used for reduction of high dynamic loads at extreme situations [6].

2.2 Supports with Hydrodynamic damper and flexible element

On the contrary to the previous supports type these supports can transmit significant axial loads from a thrust ball-bearing to its casing. They can be located in a compressor or turbine area (Fig. 3).

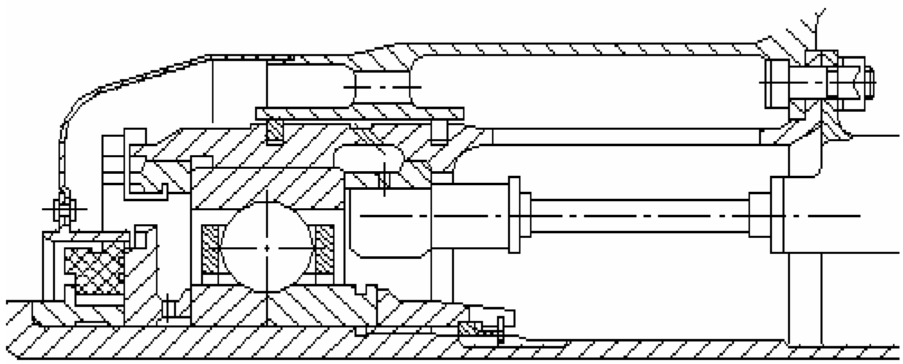

Fig. 3 A damper support with a flexible element.

The flexible element is a metal hollow cylinder having axial slots cut along circumference to permit flexibility and resiliency ("squirrel - cage"). One end the cylinder is firmly affixed to a casing, the other end firmly affixed to end of bearing retainer. Between the retainer outer surface and the casing there is an annual clearance δ, Fig.12 .

The bearing radial load produces bending displacements in the bars of damper flexible element. Every bar operates as a stiffly restrained by its ends beam, one of the beam ends can move under the load. All the beams have equal displacements limited by the radial clearance δ. The bearing axe moves parallel to itself so the bearing misalignments and local overloadings are excluded. Such supports are used both with annular and in angular contact bearings.

For the bars to have equal stiffness in all the bending directions they are to have circular cross-section. There are some designs where the circular bars are brazed or thread inserted into flanges. Some designs have bars of rectangular cross-section.

Damping function is performed by a 0.1 to 0.3 mm thick oil film. The oil film thickness determines the support radial displacement. The oil is high pressure supplied into the spacing from a circular groove. Damping performances depend on thickness and length of the oil film.

The damper dimensions are approximately determined by calculation and tuned experimentally.

A total support stiffness consists of a flexible element stiffness k_y and oil film stiffness k_g

$$k_{\Sigma} = k_{y} + k_{g}$$

Damper supports with flexible elements are designed in terms of having the oil film stiffness at least one order smaller than the flexible element stiffness. If so, the support dynamic performance is practically constant (Fig. 4).

This type of supports advantages are the following: constant stiffness performances within the whole operating range; possibility to transmit high axial loads; reliability; high damping; a rotor centering without rotation; there are available calculating models.

The main shortcomings are large sizes and mass and nonlinear damping performance.

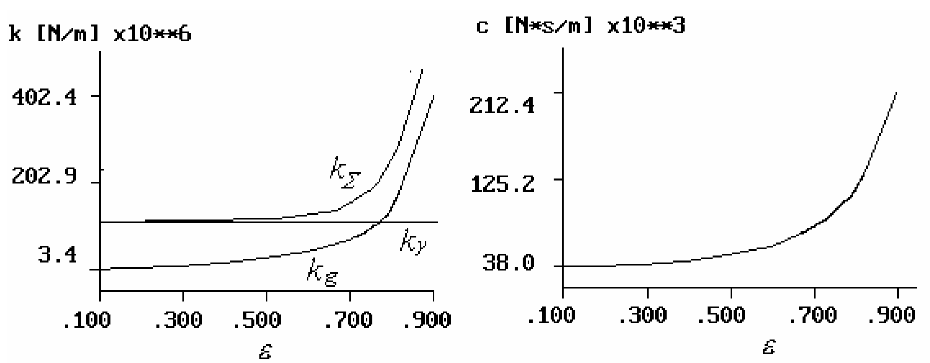

Fig 4 Stiffness and damping of a damper support with a flexible element.

2.3 A damper support with a flexible ring ("Allison" ring).

Damper supports with flexible rings (Fig.5, Fig A4.1) are mostly used in military engines for maneuvrable planes with high overloadings.

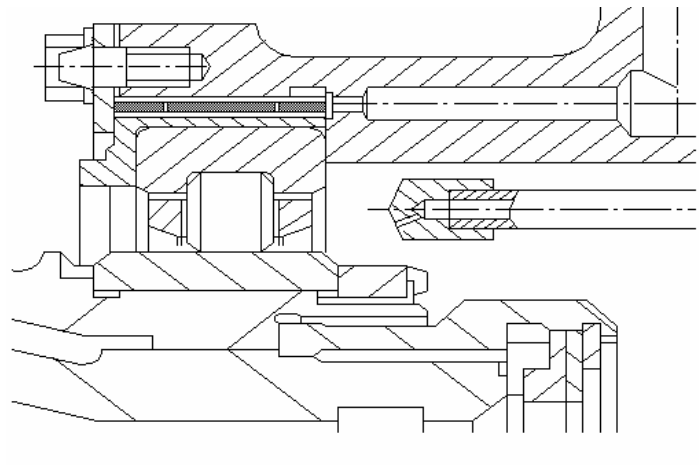

Fig. 5 A damper support with a flexible ring.

The main part of this support is a thin-wall ring (Fig. 6) installed into a clearance between the bearing outer ring and the casing.

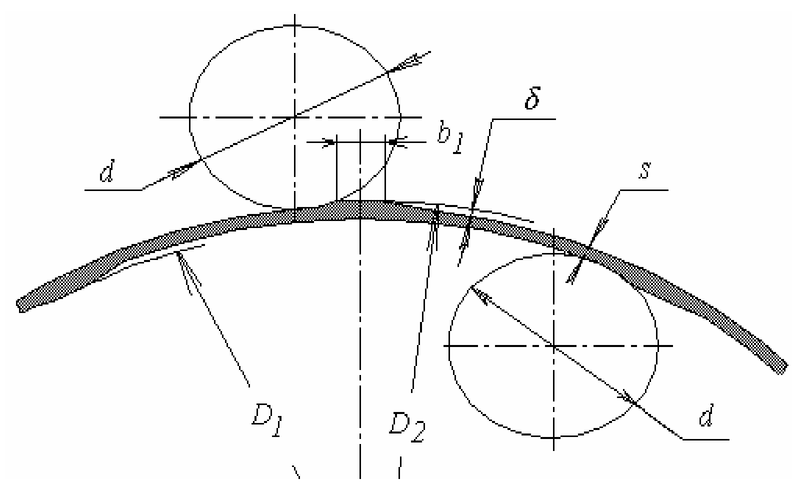

Fig. 6 A flexible ring with pedestals.

Dimensions D_1 and D_2 are determined by standards. b_1 is defined by standart, δ, s -are determined by analysis and test, d - cutting mill diameter.

The ring has center pedestals staggered on outer and inner cylinder of the ring. The center pedestals together with other elements form chambers, where is the oil supplied. Number of the center pedestals and their dimensions are regulated by a standard. Usually the pedestals height is 0.15 to 0.3 mm which limits maximal bending of the ring.

The bearing radial load bends each of the ring parts located between the pedestals. Oil is pressed from chamber to chamber through tip clearances and small holes in the ring, thus producing the damping effect.

Examples of engine insrtalled ring dimensions are given in table below.

Table 1

№	Engine	D_1 mm	D_2 mm	Number of pedestals	b_1 mm	δ mm	d mm	Material
1	АИ-24	96	93	6	6	0.12	30	60C2A
2	АИ-25	137	134	10	6	0.15	30	60C2C
3	ТВ3-117	136	133.2	12	5	0.2	30	40XHMA

Sometimes the support flexibility is increased by settling of two co-axial rings (Fig. 7).

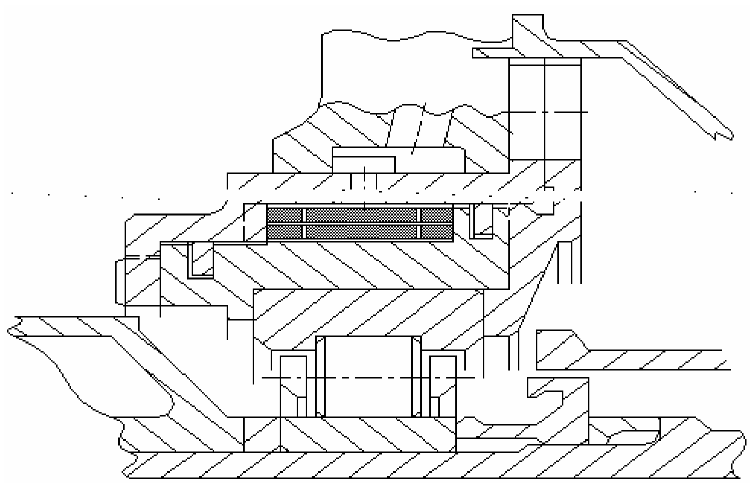

Fig. 7 A damper support with two rings

This type of supports has the following advantages: small size and mass; linear stiffness characteristics (Fig.8); rotor centring; availability of a standard which permits to determine the ring dimensions.

The support shortcomings are: high requirements to manufacturing accuracy; lack of mathematical models of the support damping*, which leads to necessity of engine test facility development.*

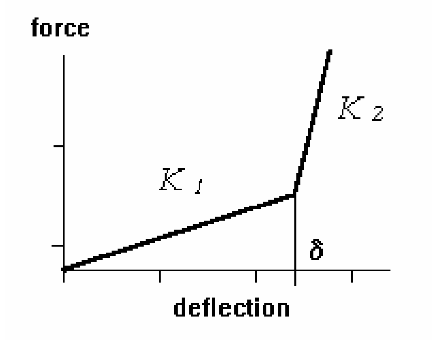

Fig 8. Stiffness of a flexible ring support.

K_1 - ring stiffness; K_2 - casing stiffness.

3. MAIN STAGES OF DAMPER SUPPORTS DESIGN

3.1 Topics

As noted above the preliminary design stage always includes a crytical speeds and mode shapes analysis. If a critical speed is within the engine operating range a frequency tuning is performed.

One of the most effective methods of critical speeds reduction is using of flexible elements in supports, which moves the critical speeds down from the operating range. But this arises a problem of passing the critical speeds when accelerating to the operating speed. Using of specific methods can help avoiding of dangerous vibrations.

On the other hand the vibrations can be caused by large rotor unbalance at the operating speed. If so the vibrations can be reduced by damping in supports.

Actual damping supports solve both problems - tuning and damping.

The support design process can be separated into the following stages based on parametric studies of the rotor dynamics analysis:

- Calculation of dangerous speeds.
- Taking of a decision on damper support using.
- Calculation of required support flexibility and damping.
- Choice of the support type.

3.2 Design of a Hydrodynamic damper support

3.2.1 Topics

Hydrodynamic damper (HD) principles can be investigated in consideration of main forces acting (Fig. 9).

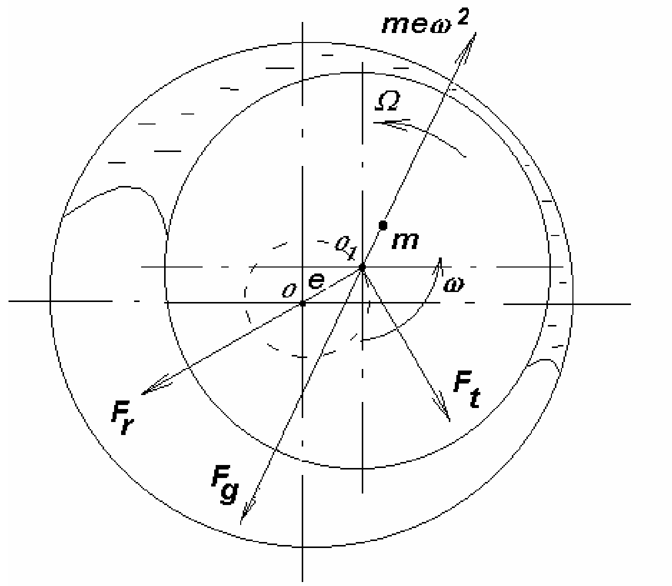

Fig 9. Forces in a Hydrodynamic damper.

$me\omega^2$ - rotor unbalance force; F_g - hydrodynamic force; F_g - bearing radial force; F_t- damping tangential force.

A HD starts operating only from some speed of rotation. At smaller speed the shaft journal lies on the inner bearing surface and the rotor is not centred.

At rotation the rotor starts precession within the clearance due to the unbalance loads. Large precession speed could produce negative pressures in the oil film, but actual liquids cannot exist at negative pressures, so the oil film loses its solidity, there develop cavities filled with oil vapour. An annular pressure distribution becomes non-uniform, which creates a hydrodynamic force. The force radial component determines the HD bearing ability, the circular component - its damping performance.

The greater is the cavitating region the larger is the hydrodymamic force and the bearing ability. The cavitating region is influenced by the rotor eccentricity and oil supply pressure. The pressure increase reduces the cavitating region which is mostly observed at small speeds of precession and small eccentricities.

It is worth mentioning that if the oil film completely fills the clearance with no cavitation the HD loses its bearing ability but has remarkable damping.

3.2.2 Evaluation of HD effectiveness by a set of non-dimensional parameters

Usually HDs are designed for operating wihin high dynamic loads range. It can be a range of a critical speed ω_c or an operating speed ω where unbalance loads reach maximal values.

The main HD dimensions which control its damping and stiffness are the damper radius, length and clearance. The damper performances can be approximately evaluated by a set of generalized parameters [2,4].

Bearing parameter *B*:

$$B = \frac{\mu \cdot R \cdot L_R^3}{m_B \cdot \omega_C \cdot \delta^3} \quad ,$$

here

μ – lubricant absolut viscosity, Pa·sec; R - bearing radius, m; L_R – squeeze film bearing land length, m; m_B – mass lumped at either of the bearing stations, кг; ω_C – pin-pin critical speed, s^{-1}; δ - radial clearance (housing radius - journal radius) , m.

The damper reduced length is calculated for a damper with annular groove:

- for a damper without end seals

$$L_R = \sqrt[3]{\sum L_i^3} \; .$$

L_i - is a land-width beetween end side and annular groove

- for a damper with end seals, for example with piston ring seals

$$L_R = \sqrt[3]{\sum \left(1{,}58 L_i\right)^3}$$

Here L_i - is a land-width beetween ring seals and annular groove i.e. land - width of the operating area.

Gravity parameter - $\overline{W}$

$$\overline{W} = (W_B / m_B \delta \omega^2) \; , \qquad \text{где} \;\; W_B = m_B \, g;$$

Unbalance parameter - U

$$U = (F_u / m_D\delta\omega^2) = e_u/\delta,$$

где F_U - unbalance force $(m_D e_U \omega^2)$; m_D mass lumped at the rotor mid-span ; e_U - unbalance eccentricity.

Frequency ratio - Ω

$$\Omega = \omega/\omega_C$$

Mass ratio - α

$$\alpha = m_B/m_D$$

For most practical tasks usually only B and W parameters are analysed, the others are not verifyed.

Below are given some results of experimental and analytical investigations of a large number of dampers [3]. The typical set of the experimental results is listed in Table 2.

Table 2

Test	δ, mm	L_R mm	Oil Type	Average viscosity Ns/m^2	Unbalance gm cm	Maximum Peak-to peak Amplitudes G, mm	Speed at which Max Amplitude occurs, rpm
A	0	0	SAE 30	35.98x10^{-2}	1.016	0.289	7248
B	0	0	SAE 30	30.10x10-^{2}	2.032	0.502	7243
1	0.0635	7.62	SAE 30	30.10x10-^{2}	5.67	0.552	6950
2	0.0635	7.62	Turbine 3	2.36x10-^{2}	5.67	0.254	6980
3	0.1905	15.24	SAE 30	32.39x10-^{2}	17	0.590	7005
4	0.1270	7.62	Turbine 3	3.13x10-^{2}	11.33	0.178	7008
5	0.1905	15.24	Turbine 3	2.25x10-^{2}	17	0.266	7240

Calculated values of the appropriate non-dimensional values are listed in Table 3, below.

Table 3

Тест.	Max. Amplitude Ratio G/δ	Unbalance parameter U	Bearing parameter B	Gravity parameter $\overline{W}$	Mass ratio α
1	8.7	0.1	8.731	0.268	0.445
2	4.1	0.1	0.684	0.268	0.445
3	3.1	0.1	2.784	0.089	0.445
4	1.4	0.1	0.114	0.134	0.445
5	1.4	0.1	0.193	0.089	0.445

Analysis and tests of modelling rotors have shown that the bearing parameter should be kept within range of $0.05<B<4$ and for the best results to a value of about 0.1. The gravity parameter W should be kept to a value less than 0.1. Values $B < 0.05$ produce rotor unstability, at $B>4$ the dynamic transmissibility t is near to 1.

Investigations of more than 20 dynamically developed engines give the following results ($\overline{W}$ parameter here is not considered):

- HDs with ring seals have $0.02<B<0.2$,
- HDs without ring seals have $0.002<B<0.8$ (note much large dispersion).

Considering these values in terms of design usage can be summarised as following:

- not all of the investigated dampers can be considered as optimal;
- good results in vibration amplitude can be obtained within a wide range of B values;
- the dimensions obtained by the non-dimensional parameters can be made more accurate by test and/or analysis.

3.2.3 Evaluation of HD stiffness and damping.

HD dynamic performances can be more accurately evaluated out of hydrodynamic lubrication theory [Ref. 5]. An isoviscous flow in the clearance is described by Reynolds lubrication equations which are resulting out of Navier-Stocks equations at simplifying assumptions:

1) the clearance size is small; 2) the flow is laminar; 3) the oil is incompressible; 4) the oil viscosity is constant; 5) the oil inertia is not considered.

The Reynolds equation is

$$\frac{\partial}{\partial x}(h^3\frac{\partial P}{\partial x})+\frac{\partial}{\partial z}(h^3\frac{\partial P}{\partial z})=6\mu\frac{\partial}{\partial x}(hu)+12\mu\frac{\partial h}{\partial t}$$

Here $x = R\theta$ - curcumferential coordinate; $u = R(\omega_1 - \omega_2)$ - sliding velocity (as usual the damper does not rotate so u=0); *h* - local oil film thickness.

The equation describes a 2-dimensional flow in axial and circular directions. More accurate equations consider oil inertia, turbulent flow etc, but they need significant computation expences so here are considered only the most influencing factors.

The relationship between the damper parameters and system inertial coordinates (Fig.10) is described by dependencies

$$\bar{e} = (X_R - X_S)\hat{i} + (Y_R - Y_S)\hat{j}$$

$$\dot{\bar{e}} = (\dot{X}_R - \dot{X}_S)\hat{i} + (\dot{Y}_R - \dot{Y}_S)\hat{j}$$

then

$$h = \delta - \bar{e}\cdot\hat{n} = \delta - (X_R - X_S)Cos\theta - (Y_R - Y_S)Sin\theta$$

$$\frac{dh}{dx} = \frac{1}{R}\frac{\partial h}{\partial \theta} = \frac{1}{R}\left[(X_R - X_S)Sin\theta - (Y_R - Y_S)Cos\theta\right]$$

and

$$\frac{dh}{dt} = -(\dot{X}_R - \dot{X}_S)Cos\theta - (\dot{Y}_R - \dot{Y}_S)Sin\theta$$

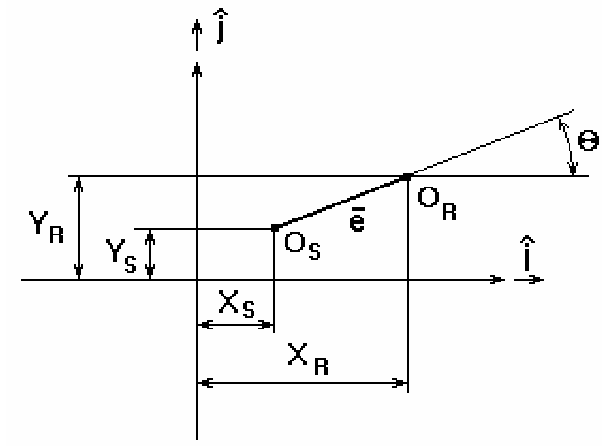

Fig.10 Inertial coordinates

It is of use here to introduce such terms as "short" and "long" dampers similar to "short" and "long" sliding bearings. The Reynolds equation is solved in these terms.

"Long" damper.

For this case the axial pressure distribution is assumed as a parabola producing ends pressure 0.75 of the middle pressure. The pressure function is

$$P(\theta, z) = P(\theta, 0) \cdot (1 - \frac{z^2}{L^2})$$

“Short" damper

If the end cavitating regions are absent the pressure distribution can be assumed as

$$P(\theta, z) = P(\theta, 0) \cdot (1 - \frac{4z^2}{L^2})$$

These assumptions at u=0 produce out of the Reynolds equations for a "long" damper

$$\frac{d}{dx}(h^3 \frac{dP}{dx}) - \frac{2h^3}{L^2} P(\theta, 0) = 12\mu \frac{dh}{dt}$$

and for a "short" damper

$$\frac{d}{dx}(h^3 \frac{dP}{dx}) - \frac{8h^3}{L^2} P(\theta, 0) = 12\mu \frac{dh}{dt}$$

These equations can be solved by a finite differences method (FDM). Derivation of the equations above gives

$$h^3 \frac{d^2P}{dx^2} + 3h^2 \frac{dh}{dx}\frac{dP}{dx} = 12\mu \frac{dh}{dt} + 2\frac{P(\theta,0)h^3}{L^2}$$

$$h^3 \frac{d^2P}{dx^2} + 3h^2 \frac{dh}{dx}\frac{dP}{dx} = 12\mu \frac{dh}{dt} + 8\frac{P(\theta,0)h^3}{L^2}$$

Change of the first and second pressure derivatives to central differences results

$$(\frac{dP}{dx})_{i,k} = \frac{P_{i,k} - P_{i-1,k}}{2\Delta x}$$

$$(\frac{d^2P}{dx^2})_{i,k} = \frac{P_{i+1,k} - 2P_{i,k} + P_{i-1,k}}{\Delta x^2}$$

and

$$P_{i-1}(\frac{h_i^3}{\Delta x^2} - \frac{3h_i^2}{2\Delta x}\frac{dh_i}{dx}) + P_i(-\frac{2h_i^3}{\Delta x^2} - \frac{2h_i^3}{L^2}) + P_{i+1}(\frac{h_i^3}{\Delta x^2} + \frac{3h_i^2}{2\Delta x}\frac{dh_i}{dx}) = 12\mu\frac{dh}{dt}$$

$$P_{i-1}(\frac{h_i^3}{\Delta x^2} - \frac{3h_i^2}{2\Delta x}\frac{dh_i}{dx}) + P_i(-\frac{2h_i^3}{\Delta x^2} - \frac{8h_i^3}{L^2}) + P_{i+1}(\frac{h_i^3}{\Delta x^2} + \frac{3h_i^2}{2\Delta x}\frac{dh_i}{dx}) = 12\mu\frac{dh}{dt}$$

Rewriting of this equation in each knot of the network in a matrix form results with a system which can be solved under boundary conditions in the oil supply areas [5, 16].

There are various available models for determination of cavitating regions in the damper . Here we use Reynolds boundary conditions:

$$\begin{cases} \frac{dP}{dx} = 0 \\ P = P_{vapour} \end{cases}$$

which can be rewritten in FDM knots terms:

$$P_{i,k} = P_{vap}, \quad \text{если} \quad P_{i,k} < P_{vap};$$

<u>Hydrodynamic forces and dynamic coefficients</u>

On the inner damper surface the forces can be determined by integration of the pressure distribution function. The X and Y force components can be determined:

$$F_x = -\int_{-L/2}^{L/2} \int_0^{2\pi} P(\theta, z) Cos\theta \cdot Rd\theta dz$$

$$F_y = -\int_{-L/2}^{L/2} \int_0^{2\pi} P(\theta, z) Sin\theta \cdot Rd\theta dz$$

If the pressure distribution within the oil film is known the integration can be performed numerically by Sympson method. The expressions define the forces applied to the rotor. Force gradients or current values of damping and stiffness coefficients are determined:

$$\left[C_{ij}\right]_{2\times 2} = \left[-\frac{\partial F_i}{\partial X_j}\right]; \left[K_{ij}\right]_{2\times 2} = \left[-\frac{\partial F_i}{\partial X_j}\right]$$

For circular precession the stiffness and damping coefficients can be calculated as following:

$$K_o = -\frac{F_r}{\bar{e}} \; ; \; C_o = -\frac{F_\tau}{\dot{\bar{e}}}$$

Here F_r and F_τ - radial and tangential components of the hydrodynamical force;

Transmision from unmovable coordinates (X, Y) to rotating coordinates *(r, τ)* is performed by an orthogonal transform:

$$\begin{Bmatrix} F_r \\ F_\tau \end{Bmatrix} = \begin{bmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{bmatrix} \begin{Bmatrix} F_x \\ F_y \end{Bmatrix}$$

3.2.4 Approximate models of the damper performances

Sometimes the approximate models are useful for calculating the damper performances [6]. These models can be obtained from analytical solutions of the Reynolds equation at additional assumptions:

1. The damper journal whirls within the spacing by circular orbits. If the damper design provides weight balancing the rotor wirls around the bearing axis, in other cases this assumption produces some uncertainty.

2. The oil pressure distribution is symmetrical related to middle line of the damper active surface. For a "short" damper the pressure distribution is approximated with a parabola, for a "long" one - as constant.

3. The cavitation film model is a π - film or 2π - film which are used for cavitation models. The first suits to cavitation zone is half of the curcle long, the second suits to absence of cavitation zone.

Calculation of the oil pressure in a sealed damper is performed for a "long" damper, for a not sealed one - as for a "short".

The damper stiffness *K* and damping *C* can be calculated out of pressure distribution for each eccentricity. Solutions [6] are given in the table 4.

Table 4

Type of film	"Short" damper	
	K	C
π – film	$\frac{RL^3\mu\omega}{\delta^3}\cdot\frac{2\varepsilon}{(1-\varepsilon^2)^2}$	$\frac{RL^3\mu}{2\delta^3}\cdot\frac{\pi}{(1-\varepsilon^2)^{3/2}}$
2π - film	0	$\frac{RL^3\mu}{\delta^3}\cdot\frac{\pi}{(1-\varepsilon^2)^{3/2}}$
Type of film	"Long" damper	
	K	C
π – film	$\frac{R^3L\mu\omega}{\delta^3}\cdot\frac{24\varepsilon}{(2+\varepsilon^2)(1-\varepsilon^2)}$	$\frac{R^3L\mu}{\delta^3}\cdot\frac{12\pi}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}$
2π - film	0	$\frac{R^3L\mu}{\delta^3}\cdot\frac{24\pi}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}$

The "short" and "long" dampers solutions can be used for calculation of dampers performances at different boundary conditions.

Fig 11 illustrates some dampers layouts that can be calculated by the described algorythms. The layouts can be separated by two main features:

- presence of end seals,
- way of oil supply.

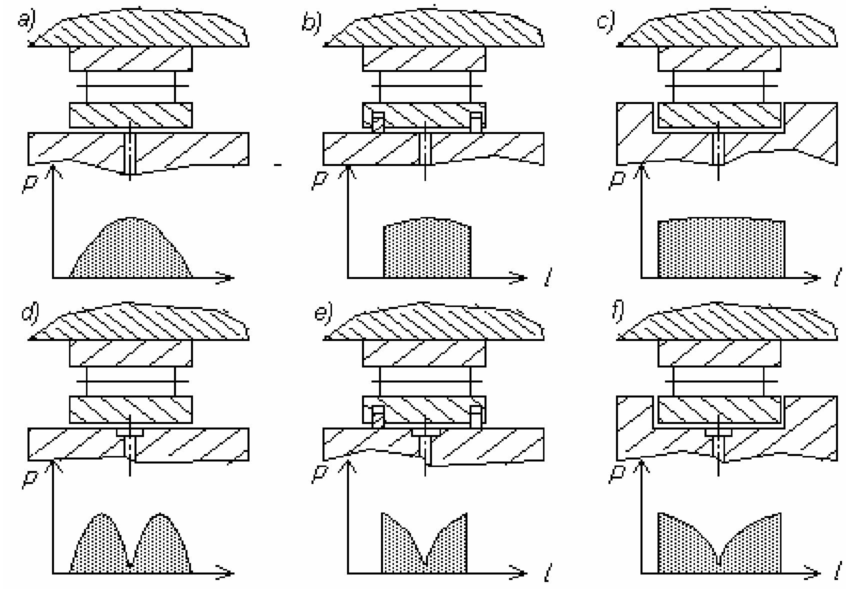

Fig 11. HDs layouts and oil pressure distributions

Version (a) is a damper without the end seals. The oil is supplied through one or a few orifices, the pressure distribution is described by a parabola.

Versions (b) and (c) have end cavitating regions, so the oil flow through the cavitating regions is not taken into consideration, the oil flow can be assumed as circular and the pressure as constant.

Versions (d), (e) and (f) differ from the upper ones by circular oil supplying grooves. Here the damper operating length is separated into two parts, each part having a distribution depending on cavitating regions. The table below gives combinations of features that permit to choose a calculating model for a particular layout, to choose a modelling factor to correct the hydraulic force and the length. For example a damper with a seal at the left end and witout seal at the right end one can be calculated by a "short" damper model with a doubled length and modelling factor 0.5.

Layout	Type of damper	Modelling factor	Length of damper
a	short	*1*	L
b	long	*1*	L
c	long	*1*	L
d	short	*2*	$L/2$
e	short	*1*	L
f	short	*1*	L

Some calculation results confirmed by tests can be summarised as the following [3, 4, 6]:

- HDs up to 0.4... 0.5 eccentricity have constant damping coefficients;
- Cavitation zone area can be reduced by increase of the oil supply pressure;
- At uncavitied oil film under circular orbits and constant eccentricity the hydrodynamic stiffness diminishes.
- Damping in an uncavited damper (2π - film) is twice larger than in a cavitated damper (π - film).
- An uncavited HD needs an additional flexible element (a “squirrel-cage”) to bear radial loads.
- Completely cavitated HDs with unbalance parameters >0.5 always had dynamic transmissibility >1.

An uncentralised HD has a considerable nonlinearity, so final determination of its dimensions can be performed only by transient analysis including a rotor.

Final investigation of a HD performance and workability is to be performed in full-size test rigs or by engine tests.

3.2.5 Matching of a damper performances to a rotor system

HD stiffness and damping depend on rotor displacements in clearance so it is nesessary to match the rotor and HD dynamic performances as to apply the parametric analysis stiffness and damping coefficients. The calculation milestone points are given below.

1) Stiffness and damping are calculated at preliminary determined damper dimensions (length, diameter, clearance and eccentricity ε_0 = 0.4 to 0.5. Desired values are near to the values obtained at parametric study.

2) At given unbalance distribution the rotor loads and displacements are calculated. Damper displacements determine eccentricity ε_1.

3) The obtained ε_1 value is compared to preliminary taken ε_0. If the difference is considerable the initial ε_0 value is changed and the (1) and (2) calculations are repeated until coinsidence of ε_0 and ε_1. The obtained values of coefficients, displacements and loads are used in further calculations.

4) Similar calculations are performed at other values of the damper clearances. The result is relation of the HD stiffness and damping by the clearance. This relation permits to determine the optimal damper clearance by optimal stiffness and damping obtained earlier in parametric studies.

5) If the optimal values cannot be reached the HD dimensions (diameter and/or length) are to be changed. There is a chance to meet a situation when the displacement and load requirements cannot be fulfilled. In this ocasion the damper type is to be changed.

NOTE: Matching procedure can be used only for centralised dampers

3.3 Design and analysis of a support with a Hydrodynamic damper and flexible element.

3.3.1 Topics.

In a support furnished with a Hydrodynamic Damper and flexible Element (HDFE) the oil film has low bearing ability at small eccentricities, in other words the FE stiffness is much higher than the film stiffness. It permits to have constant support stiffness and to obtain high damping in a not cavitated damper.

Oil film stiffness and damping can be calculated equally to a HD without an EE.

Below are given some specific features of a HDFE:

- If the amplitude of the journal orbit is very large the oil film hydrodynamic stiffness can be greater than the FE stiffness.
- If the amplitude of the journal orbit is about half of the clearance the FE centers the damper and reduces dynamic transmissibility.
- A single-directed dynamic loading increases the dynamic transmissibility coefficient and causes a subharmonic movement due to nonlinear effects.
- At some level of unbalances the vibration amplitude can change rapidly ("jump" - effect). At speed increase the amplitude rapidly increases, at speed reduction the amplitude can rapidly decrease. This effect is concerned to a rapid change of the dynamic transmissibility coefficient due to nonlinear damper properties.
- A HDFE design improvement can cause increase of transmitted forces even compared to stiff supports. It can occur in a range of speeds $\Omega <$ 1.4, which should be avoided by support design.

3.3.2 Flexible element analysis

The flexible element layout and calculating scheme is shown in Fig 12.

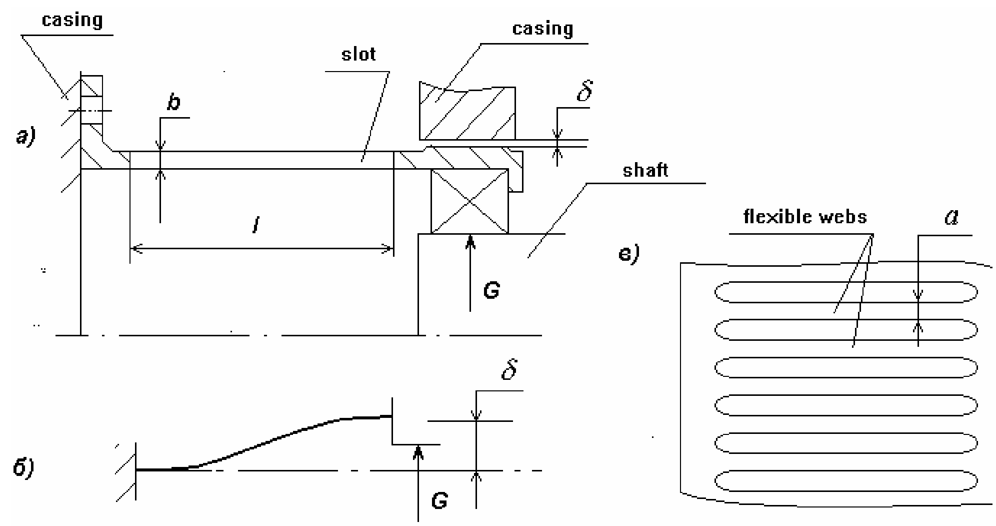

Fig 12 . A “squirrel-weel” type flexible element.

a) design scheme and main dimensions; b) loaded bar displacements;

c) cylinder involute view

An FE stiffness can be calculated by

$$K = \frac{nEab(a^2 + kb^2)}{2l^3},$$

where

n - number of bars;

a, b, l - width, thickness and length of a bar accordingly;

E - Young module of the bar material at operating temperature.

$k = \frac{1}{\left(1 + \frac{2\sqrt{ab}}{l}\right)^3}$ - correction coefficient, depending on the flexible web dimensions

Maximal alterating stress in the bar

$$\sigma_d = \frac{3E\delta}{l^2}\left(k^{\frac{2}{3}} b\cos\varphi + a\sin\varphi\right),$$

where

$$\varphi = \arctan\frac{a}{bk^{\frac{2}{3}}} + n\pi; \quad n = 0 \text{ or } 1$$

δ - radial clearance

Static displacement under the support weight G loading

$$\delta_0 = G/K \quad .$$

Static stress in a bar under the weight loading

$$\sigma_{\delta_0} = \sigma_d \frac{\delta_0}{\delta} \quad .$$

To balance the weight displacement as to center the bearing its initial location is to be preliminary moved δ_0 upwards.

Fatigue margin is determined by dynamic σ_α and static σ_m stresses

$$n_\sigma = \frac{\sigma_{\neg-1} - \psi_\sigma \sigma_m}{\left(k_\sigma\right)_d \sigma_\alpha},$$

where

$\sigma_{\neg-1} = 0.85\sigma_{-1}$ - fatigue limit of a flat coupon;

σ_{-1} – - fatigue limit of a standard circle coupon;

ψ_σ- material sensitivity to the fatigue cycle non-symmetry;

$\left(k_\sigma\right)_d = \dfrac{k_\sigma + k_\sigma^n - 1}{\varepsilon_\sigma}$ - fatigue resistance coefficient;

k_σ - effecttive stress concentration;

k_σ^n - surface state coefficient;

ε_σ - scaling factor.

Sometimes fatigue and stiffness requirements contradict each other. If so a "two-stores" design can be recommended which is a support furnished with two co-axial “squirrel-cages”.

3.4 Design and analysis of a support with a flexible ring

3.4.1 Topics

We assume here that by this stage the engine dynamics parametric study is completed, needed stiffnesses, damping and flexible support displacements δ_0 are determined.

The damper support design is to consider some specific details:

1) This manual permits a choice of the flexible ring dimensions meeting the flexibility and strength requirements.

2) There are no available methods of damping calculation, or determining needed throttle holes diameters and number.

3) The damping support development and tuning should be preliminary performed in a test rig.

4) Final checking of the support workability is an engine test exclusively.

3.4.2 Choice of the damper main dimensions

1) The outer diameters of the ring (Fig. 6) are taken according to a standard (see Appendix 3). The diameters are taken by the bearing outer diameter considering that the smooth ring surface is fitted on the bearing outer ring. There are possible designs where the ring is located apart from the bearing. If so the diameters are determined by the design considerations.

2) The flexible ring width *b* is taken by design considerations. Usually it is a little smaller than the bearing width.

3) The center pedestals number *n* is taken according to the Table A3.1 above under as limited by permittable stresses at given flexibility and displacement.

4) Taken ring dimensions are used for final calculation of the ring flexibility and stresses.

5) The ring is installed into a support casing on a smooth ring with a transition fit. The tolerances are 5-th qualitate for basic shaft and 6-th for basic hole. The maximal displacement is checked at ultimate operating temperature conditions. If a ring is installed with an interference fit its flexibility is to be investigated by a test.

6) Flexible rings sometimes have special elements to avoid the ring turning.

3.4.3 Calculation of a ring flexibility

A ring flexibility can be evaluated by the following formulae (see Fig.6):

$$\alpha = \frac{(D_{av} - 0.3 \cdot b_1 \cdot n)^3}{0.129 b E n^4 s^3}\left[1-\left(1-\frac{s^3}{s_b^3}\right)\left(1.45A - 0.9A^2 + 0.2A^3\right)\right],$$

where

$$D_{av} = \frac{D_2 + D_1}{2};$$

D_1, D_2 - ring inner and outer diameters;

b - ring width;

s - ring thickness.

$$s = \frac{D_2 + D_1}{2} - 2\delta \;;$$

δ – maximal displacement (equal to a center pedestal height);

$$s_b = s + \delta \;;$$

E - Young module;

n - number of center pedestals ;

$$A = \frac{\left(b_1 + \sqrt{d\delta}\right)n}{D_{av}}$$

d - milling cutter diameter;

b_1 - center pedestal width.

Stress in the ring is calculated by the formulae

$$\sigma = 1.1 E S_b \left(\frac{n}{D_{cp}}\right)^2 \delta$$

The fatigue safety factor is calculated by a formulae similar to a flexible element one

$$n_\sigma = \frac{\sigma_{\neg-1} - \psi_\sigma \sigma_m}{\left(k_\sigma\right)_d \sigma_\alpha}$$

4. EXAMPLES OF DIFFERENT DAMPER SUPPORTS DESIGN

4.1 Design of a HD support

Initial Data:

- The damper has piston-type ring seals, uncentralised;
- The damper diameter D=150 mm
- Type of oil MK-22 - dynamic viscosity at 100 ^{O}C μ=2.05x10^{-2} Ns/m^2
- Required support stiffness K = 0.8x10^7 N/m;
- Damping C > 15000 Ns/m;
- The rotor operating speed (tuning speed) ω = 800 s^{-1};
- Mass lumped at either of the bearing stations m_B = 50 kg

1. Determination of an optimal damper clearance δ by recommended gravity parameter.

Assume a value $\overline{W}$ =0.1. Because of $W_B = m_B g$ the clearance

$$\delta = \frac{W_B}{m_B \omega^2 \overline{W}} = (50x9.81)/(50x800^2x0.1)=$$

0.000153 м = 0.153 mm

2. The parameter B=0.1 value determines the damper length L_R.

$$B = \frac{\mu \cdot R \cdot L_R^3}{m_B \cdot \omega_C \cdot \delta^3}$$

$$L_R = \sqrt[3]{\frac{B \cdot m_B \cdot \omega_C \cdot \delta^3}{\mu \cdot R}} = \sqrt[3]{\frac{0.1 \cdot 50 \cdot 800 \cdot (0.153 \cdot 10^{-3})^3}{2.05 \cdot 10^{-2} \cdot 0.075}} =$$

= 0.021 м = 21 mm

3. We choose a sealed damper with oil supplying holes, Fig.11b.

Then the damper can be considered as a "long" one and its length can be estimated as $L = L_R$..

4. The damper performances can be evaluated by approximate dependencies. The analysed damper is not centered, so it is to have some stiffness or bearing ability. So the "π-film" boundary conditions are considered.

For an eccentricity value ε=0.4

$$K = \frac{R^3 L\mu\omega}{\delta^3}\cdot\frac{24\varepsilon}{(2+\varepsilon^2)(1-\varepsilon^2)} =$$

$$\frac{0.075^3\cdot 0.021\cdot 2.05\cdot 10^{-2}\cdot 800}{(0.153\cdot 10^{-3})^3}\cdot\frac{24\cdot 0.4}{(2+0.4^2)(1-0.4^2)} =$$

$$\underline{= 0.214\ \text{x}10^9\ \text{N/m}}$$

$$C = \frac{R^3 L\mu}{\delta^3}\cdot\frac{12\pi}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}} =$$

$$\frac{0.075^3\cdot 0.021\cdot 2.05\cdot 10^{-2}}{(0.153\cdot 10^{-3})^3}\cdot\frac{12\cdot 3.14}{(2+0.4^2)(1-0.4^2)^{\frac{1}{2}}} =$$

$$\underline{= 0.965\text{x}10^6\ \text{Ns/m}}$$

The obtained values of stiffness and damping are much higher than required (about two orders). So it is impossible to fulful the initial data requirements within the chosen scheme.

5. Another damper scheme is a sealed damper with a central groove, Fig.11e. This damper can be considered as "short" π-film damper with a reduced length $L = L_R$

At eccentricity $\varepsilon = 0.4$

$$K = \frac{RL^3\mu\omega}{\delta^3}\cdot\frac{2\varepsilon}{(1-\varepsilon^2)^2} =$$

$$\frac{0.075\cdot 0.021^3\cdot 2.05\cdot 10^{-2}\cdot 800}{(0.153\cdot 10^{-3})^3}\cdot\frac{2\cdot 0.4}{(1-0.4^2)^2} = \quad \underline{0.224\ \text{x}\ 10^7\ \text{N/m}}$$

$$C = \frac{RL^3\mu}{\delta^3}\cdot\frac{\pi}{(1-\varepsilon^2)^{3/2}} =$$

$$= \frac{0.075\cdot 0.021^3\cdot 2.05\cdot 10^{-2}}{2\cdot(0.153\cdot 10^{-3})^3}\cdot\frac{3.14}{(1-0.4^2)^{\frac{3}{2}}} =$$

$$= \underline{0.811\text{x}10^4 \text{ Ns/m}}$$

6. To meet initial requirements on stiffness and damping it is possible to increase the damper length in 1.5 times.

NOTE: This calculation is based on Reynolds equations solution for centralised dampers (the rotor weight neglected).

7. If there is available code for nonlinear dynamics analysis it is of use to repeat the calculation at different loading conditions [11].

8. The damper performances are to be checked by testing in a special facility.

4.2 Design of a HDFE

HDFE design can be separated into two stages: HD design and Flexible element design. Below are given two examples of such design.

EXAMPLE 1

Design of a HD with a "squirrel-cage" type flexible element.

<u>*Input data*</u>

- Total support stiffness K = 0.4x10^8 N/m;
- Damping C > 10000 Ns/m;
- Operating speed n = 14000 rpm = 1465 s^{-1} ;
- Oil viscosity μ = 0.266x10^{-2} Ns/m^2;
- Lumped bearing mass m_B = 33.43 kg;

- The damper diameter D = 130 mm.

1. Defermination of an optimal damper clearance δ by recommended gravity parameter.

Considering $W_B = m_B\ g$ it is possible to assume a value $\overline{W}$ =0.1 and

$$\delta = \frac{W_B}{m_B \omega^2 \overline{W}} = (33.43 \text{ x } 9.81)/(33.43 \text{ x } 1465^2 \text{ x } 0.1)$$

0.000045 м = 0.045 mm

To match manufacturing requirements we assume δ=0.1mm.

2. The parameter B=0.1 value determines the damper length L_R:

$$B = \frac{\mu \cdot R \cdot L_R^3}{m_B \cdot \omega_C \cdot \delta^3}$$

$$L_R = \sqrt[3]{\frac{B \cdot m_B \cdot \omega_C \cdot \delta^3}{\mu \cdot R}} = \sqrt[3]{\frac{0.1 \cdot 33.43 \cdot 1465 \cdot (0.1 \cdot 10^{-3})^3}{2.66 \cdot 10^{-3} \cdot 0.0650}} =$$

= 0.0305 м = 30.5 mm

3. Calculation of the damper dynamic performances.

The damper dynamic performances (stiffness and damping) are calculated after the main dimensions are determined as to fit the requirements.

The approximate calculations use both "π-film" and "2π-film models. The centered HD is to have lower stiffness than the “squirrel-cage”

The calculations below are related to a sealed by its ends ("long" damper) with a few oil supplying holes, Fig.11b. For the ε=0.4 eccentricity the two film models give the following result:

"2π-film".

$$C = \frac{R^3 L\mu}{\delta^3} \cdot \frac{24\pi}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}} =$$

$$= \frac{0.065^3 \cdot 0.0305 \cdot 2.66 \cdot 10^{-3}}{(0.1 \cdot 10^{-3})^3} \cdot \frac{24 \cdot 3.14}{(2+0.4^2)(1-0.4^2)^{\frac{1}{2}}} =$$

$$= \underline{848250 \text{ Ns/m}}$$

$$\underline{K = 0}$$

"π-film".

$$C = \frac{R^3 L\mu}{\delta^3} \cdot \frac{12\pi}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}} =$$

$$= \frac{0.065^3 \cdot 0.0305 \cdot 2.66 \cdot 10^{-3}}{(0.1 \cdot 10^{-3})^3} \cdot \frac{12 \cdot 3.14}{(2+0.4^2)(1-0.4^2)^{\frac{1}{2}}} =$$

$$\underline{= 424126 \text{ Ns/m}}$$

$$K = \frac{R^3 L\mu\omega}{\delta^3} \cdot \frac{24\varepsilon}{(2+\varepsilon^2)(1-\varepsilon^2)} =$$

$$\frac{0.065^3 \cdot 0.0305 \cdot 2.66 \cdot 10^{-3} \cdot 1465}{(0.1 \cdot 10^{-3})^3} \cdot \frac{24 \cdot 0.4}{(2+0.4^2)(1-0.4^2)} =$$

$$\underline{= 0.173 \text{x} 10^9 \quad \text{N/m}}$$

The stiffness and damping exceed the needed values so much, that it is nesessary to consider another type of the damper.

4. Two versions are considered below

- a sealed damper with a central oil supply groove, Fig.11e;
- a not sealed damper with a few oil supplying holes, Fig 11a.

Both versions suit to a "short" damper model with reduced length L_R = 30.5 mm

Two film models are analysed at ε=0.4 eccentricity

"2π-film".

$$C = \frac{RL^3\mu}{\delta^3}\cdot\frac{\pi}{(1-\varepsilon^2)^{3/2}} =$$

$$= \frac{0.065\cdot 0.0305^3\cdot 2.66\cdot 10^{-3}}{(0.1\cdot 10^{-3})^3}\cdot\frac{3.14}{(1-0.4^2)^{\frac{3}{2}}} = \underline{20010\ \text{Ns/m}}$$

$K = 0$

π-film".

C = 10005 Ns/m

$$K = \frac{RL^3\mu\omega}{\delta^3}\cdot\frac{2\varepsilon}{(1-\varepsilon^2)^2} =$$

$$\frac{0.065\cdot 0.0305^3\cdot 2.66\cdot 10^{-3}\cdot 1465}{(0.1\cdot 10^{-3})^3}\cdot\frac{2\cdot 0.4}{(1-0.4^2)^2} =$$

$= 0.815\ \times 10^7$ N/m

For damper with the holes

$L = L_R =$30.5 mm.

For damper with the central annular groove

$L_1=L_2 =$15.25 mm

NOTE: The approximate solutions used here do not consider oil supply pressure, so the cavitating region length is not determined. The design is performed in such a manner as to meet needed performances at any cavitation state.

4. The next step is ajustment of the damper performances to the rotor dynamic system. This procedure uses stiffness parameters

obtained out of rotor dynamics analysis. Here will be obtained new performances of the dynamic system including the determined damper: suports loads, displacements, vibration velocities.

It is necessary to note that the ajustment procedure can cause a larger eccentricity value and the dynamic transmissibility can become nearer to value 1. If so, the damper length can be reduced or the clearance can be increased under the B parameter control.

5. Actual performances of the damper and rotor system can be obtained by testing of a special rig or by engine test.

6. Retain stiffness of the Flexible element

$$K_{б} = K_{\Sigma} - K = 0.4 \times 10^8 - 0.0815 \times 10^8 = \underline{0.3185 \times 10^8 \text{ N/m}}$$

EXAMPLE 2

Stiffness and stress analysis of a "squirrel - cage" type flexible element.

Initial Data:

- Weigt of rotor lumped at either of the bearing station G = 1700 N;
- Maximal radial displacement δ= 0.275 mm;
- The axial slots length and width - l = 75 mm; Ap = 5 mm;
- Flexible cylinder outer and inner diameters D_o=280 mm, D_i=271mm;
- Number of flexible bars n = 96;
- The cylinder material 18X2H4MA (Russian standard), Young module E=0.19*10e12 N/m^2;

1. Width a and height b of each bar cross - section (Fig.12)

$$a = \frac{\pi \cdot (D_o + D_i)}{2n} - Ap = \frac{3.14(280+271)}{2 \cdot 96} - 5 = \underline{4.01 \text{ mm}}$$

$$b = \frac{(D_H - D_B)}{2} = \frac{(280-271)}{2} = \underline{4.5 \text{ mm}}$$

2. Flexible element stiffness

$$k = \frac{1}{\left(1+\frac{2\sqrt{ab}}{l}\right)^3} = \frac{1}{\left(1+\frac{2\sqrt{4.01\cdot 4.5}}{75}\right)^3} = 0.724$$

$$K = \frac{nEab(a^2+kb^2)}{2l^3} = \underline{1.2\text{x}10^4 \text{ N/mm}}$$

3. Maximal alterating stress at maximal damper displacement δ=0.275 мм

$$\varphi = \arctan\frac{a}{bk^{\frac{2}{3}}} + n\pi = \arctan\frac{4.01}{4.5\cdot 0.806} = \underline{47^{\circ}48'}$$

$$\sigma_d = \frac{3E\delta}{l^2}\left(k^{\frac{2}{3}} b\cos\varphi + a\sin\varphi\right) = = \frac{3\cdot 1.9\cdot 10^5\cdot 0.275}{75^2}\left(0.724^{\frac{2}{3}}\cdot 4.5\cdot 0.6717 + 4.01\cdot 0.7524\right) =$$

151 MPa

4 . Weight displacement

$\delta_0 = G/K$ = 1700/1.2x10^4 = 0.14 mm

So the flexible bushing may be preliminary displaced 0.14mm upwards as to be centered under the weight load. Then the δ=0.275mm clearance will be uniform in circumferential direction.

5 Static stress in bars under the weight load

$$\sigma_{\delta_0} = \sigma_d\frac{\delta_0}{\delta} = 152(0.14/0.275) = \underline{77.5 \text{ MPa}}$$

6. Fatigue safety factor

The bar material fatigue performances

σ_{-1}=650 MPa; ψ_σ=0.85; k_σ =1.4; k_σ^n =1.25; ε_σ =1

$\sigma_{\neg-1} = 0.85\sigma_{-1}$ = 0.85x650=552.5 MPa

$$\left(k_\sigma\right)_d = \frac{k_\sigma + k_\sigma^n - 1}{\varepsilon_\sigma} = 1.65;$$

At estimated operating conditions

$\sigma_a = \sigma_d$ =151 MPa; $\sigma_m = \sigma_{\delta 0}$ = 77.5 Mpa

$$n_\sigma = \frac{\sigma_{\neg -1} - \psi_\sigma \sigma_m}{(k_\sigma)_d \sigma_\alpha}$$ = (552.5-0.85x77.5)/(1.65x151) = <u>1.96</u>

This value suits the fatigue resistance requirement $n_\sigma > 1.3$

4.3 Design of a damper support with a flexible ring

Initial Data:

- Bearing outer diameter D_o 60 mm;
- Required ring stiffness $k = 0.2 \times 10^7$ N/m;
- Bearing lumped mass m = 15 kg;
- Ring material 40XHMA (Russian standard), Young module E = 0.19×10^{12} N/m2;
- Overloading coefficient n = 2.5;
- Maximal unbalance of the rotor support $(me)_d$ = 20gr cm;
- Dynamic transmissibility coefficient of the shaft vibrations $k_d = 4$.

1. Radial displacement

$$e = \frac{(me)_d}{m}$$ = 20 x10^{-5} / 15 = 0.13x10^{-4} м = <u>0.013 mm</u>

$$\delta_0 = \frac{mn}{k} g + k_d e$$ = (15 x2 / 0.2x10^7) x 9.81 + 4x0.13x10^{-4} =

1471.5 x10^{-7} + 0.52x10-^{4} = 1.99 x10^{-4} м = <u>0.19 mm</u>

2. The ring inner diameter

Usually between the flexible ring and the bearing outer ring there is located a cylindrical bushing. The ring inner diameter is taken out of the table A3.1 as the nearest D_1=65mm. Then the bushing thickness is 2.5mm. The ring outer diameter is D_2=68mm.

Number of center pedestals *n*=6 is determined out of the ring displacment limit $d_0 < d_{max}$. The other dimensions are taken out of the table: center pedestals width b_1=5mm, milling cutter diameter *d*=20 mm.

3. The ring width is taken out of design consideration b=14 mm.

4. The ring thickness.

s=(D_2-D_1) /2 -2 δ_0 = 68-65) / 2-2x0.19 =1.5 - 0.38 =1.12 mm

5. The ring stiffness / flexibility

$$D_{av} = \frac{D_2 + D_1}{2} = (65 + 68)/2 = \underline{66.5\ \text{mm}}$$

$$s_b = s + \delta_0 = 1.12 + 0.19 = \underline{1.31\ \text{mm}}$$

$$A = \frac{(b_1 + \sqrt{d\delta_0})n}{D_{av}} = \frac{(5 + \sqrt{20 \cdot 0.19})6}{66.5} = \underline{0.627.}$$

$$\alpha = \frac{(D_{av} - 0.3 \cdot b_1 \cdot n)^3}{0.129bEn^4 s^3}\left[1 - \left(1 - \frac{s^3}{s_b^3}\right)\left(1.45A - 0.9A^2 + 0.2A^3\right)\right] =$$

$$= \frac{(66.5 - 0.3 \cdot 5 \cdot 6)^3}{0.129 \cdot 14 \cdot 1.9 \cdot 10^5 \cdot 6^4 \cdot 1.12^3}\left[1 - \left(1 - \frac{1.12^3}{1.31^3}\right) \cdot (1.45 \cdot 0.627 - 0.9 \cdot 0.627^2 + 0.2 \cdot 0.627^3)\right] = \underline{235\text{x}10^{-6}\ \text{mm/N}}$$

Stiffness k = $\underline{0.425 \text{ x } 10^{7}\ \text{N/m}}$

7. St:ress in the ring

$$\sigma = 1.1ES_b\left(\frac{n}{D_{cp}}\right)^2 \delta = 1.1 \cdot 1.9 \cdot 10^5 \cdot 1.31\left(\frac{6}{66.5}\right)^2 0.19 = \underline{420\ \text{MPa}}$$

8. Fatigue safety factor.

The ring material 40XHMA properties:

σ_{-1} =500MPa; ψ_σ=0.3; k_σ =1.1; k_σ^n =1.1; ε_σ =1

$$\sigma_{\neg-1} = 0.85\sigma_{-1} = 0.85\text{x}500 = 425\ \text{MPa}$$

$$\left(k_\sigma\right)_d = \frac{k_\sigma + k_\sigma^n - 1}{\varepsilon_\sigma} = 1.2;$$

The ring stress is alterating, so

$\sigma_a = \sigma_d / 2 = 210$ MPa; $\sigma_m = \sigma_d/2 = 210$ MPa

$$n_{\sigma} = \frac{\sigma_{\neg-1} - \psi_{\sigma}\sigma_{m}}{\left(k_{\sigma}\right)_{d}\sigma_{\alpha}}$$ = (425-0.3x210)/(1.2x210) = <u>1.436</u>

The fatigue safety factor is to be not smaller than 1.3 to 1.4.

The resulting stiffness is about two (2) times higher than required. It can be corrected by the ring width or by the thickness under the stress limit control.

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APPENDIX 1

Aircraft engine and turbine oil viscosity

Temperature °C	μ, Ns / m²			
	$T_п$-22	$T_п$-36	MC-20	MK-22
10	0.1890	0.3006	2.430	3.550
20	0.0855	0.1450	1.000	1.860
30	0.0475	0.0740	0.4650	0.620
40	0.0318	0.0432	0.2350	0.300
50	0.01874	0.0272	0.1310	0.164
60	0.01275	0.0179	0.0785	0.096
70	0.00905	0.0126	0.0500	0.0605
80	0.00675	0.00920	0.0339	0.0400
90	0.00513	0.00680	0.0238	0.0273
100	0.00400	0.00506	0.01725	0.0205
110	-	-	0.01305	0.0145
120	-	-	0.01010	0.01105
130	-	-	0.00805	0.00875
140	-	-	0.00650	0.00705
150	-	-	0.00538	0.00570

APPENDIX 2

Calculation of a lumped mass at either of the bearing station m_B

The mass is calculated out of equivalence of supporf loads caused by a stiff rotor and by the mass m_B.

$$P \cdot L = P_d \cdot l - M_d \text{ , где}$$

$$P_d = m_d \cdot e \cdot \frac{l}{L} \omega^2; \qquad M_d = -J_d \cdot \omega^2 \frac{e}{L};$$

m_d , J_d - disk mass and transverse moment of inertia; ω - speed of rotation.

Load P can be interpreted as a mass load reduced to the support:

$$P = m_B \cdot e \cdot \omega^2 ,$$

Then the loads equivalence gives

$$m_B \cdot e \cdot \omega^2 \cdot L = m_d \cdot e \cdot \frac{l^2}{L} \cdot \omega^2 + J_d \cdot \omega^2 \frac{e}{L}$$

Then the reduced mass can be calculated

$$m_B = m_d \cdot \frac{l^2}{L^2} + J_d \frac{1}{L^2}$$

If the rotor includes k inertia elements, their total reduced mass is

$$m_B = \sum_{i=1}^{k} m_{B_i}$$

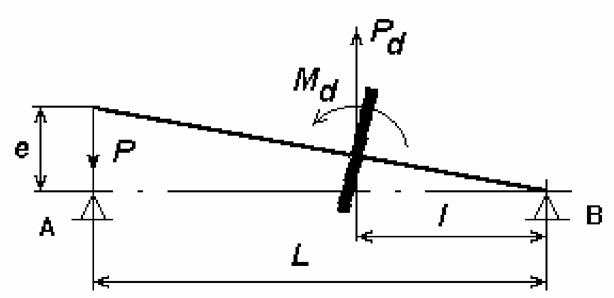

Fig. A2.1

e - orbit radius of shaft due to a support flexibility

APPENDIX 3

Flexible ring standarted dimensions

Table A3.1

D_1	D_2	Number of pedestals n	δ_{max}	b_1	d
55	58	4	0.165	5	20
		6	0.152		
58	61	4	0.120		
		6	0.170		
60	63	4	0.130		
		6	0.180		
65	68	4	0.100		
		6	0.220		
71	74	6	0.270		
		8	0.138		
75	78	6	0.313	6	30
		8	0.155		
78	81	6	0.338		
		8	0.168		
83	86	6	0.325		
		8	0.194		
88	91	6	0.300		
		8	0.223		
93	96	6	0.240		
		8	0.148		
103	106	8	0.334		
		10	0.190		

NOTES:

1. *The ring materials are steels 60C2A and 40XHMA (Russian).*
2. *The maximal displacement is limited by a maximal stress 490 MPa and flexibility 867 nm/N at the ring bended to the limit.*
3. *All the dimensions are given in mm.*