LEONTIEV M.K.
SNETKOVA E.I.

NONLINEAR MODELS OF ROLLING BEARINGS IN ROTORDYNAMICS

Information about authors

Leontiev Mikhail K., professor of Moscow Aviation Institute (National Research University); PhD. MAI, Volokolamskoe road, 4, Moscow, 125993; tel.: 8-985-768-71-29; e-mail: lemk@alfatran.com

Snetkova Ekaterina I., lecturer assistant of Moscow Aviation Institute (National Research University). MAI, Volokolamskoe road, 4, Москва, А-80, ГСП-3, 125993; тел.: 8-903-685-81-11; e-mail: lemk@alfatran.com

Abstract

The mathematical model of the nonlinear ball bearing with five freedom degrees and the results of its application are considered. The model is used in dynamic analysis of the complex rotor systems. It is built on the basis of Hertz theory. The ball bearing model with 5 freedom degrees takes into account clearance in the bearing, the number of rolling elements in loading area, contact stiffness, axial load acting on the bearing and contact angle. Results of static and dynamic analysis of the point rotor with the nonlinear bearing are given. It is proved that axial load acting on the bearing and the unbalanced rotor force influences the bearing stiffness a lot.

Key words

rolling bearings, clearance, stiffness characteristics, mathematical model

Introduction

Stiffness and damping characteristics of rolling bearings are usually estimated approximately when analyzing linear rotor systems with such bearings. As a rule, shortcoming of such approach is that in reality a bearing stiffness depends significantly on its loading by forces acting in the rotor system, i.e. on operating modes, geometry and clearances in the bearing, fits of the inner and outer rings in the rotor and stator, etc.

For example, simple estimation shows that clearance in the bearing lowers the support stiffness and resonance frequencies correspondingly [1]. The same results are given in the article [2], where unsteady behavior of the rotor supported by the ball bearings with inner clearance is investigated.

Eliminating of all these factors worsens both quantitative and qualitative results when analyzing the rotor systems [3]. Particularly, if estimation of the main rotor harmonic at frequency and amplitude may be done with accuracy that is convenient for designers of rotating machines, quantitative and qualitative analysis of the rotor system at other components of vibration signal is impossible. The task may be solved completely taking into account nonlinearity of stiffness characteristics of the rolling bearings [4], [5].

The bearings models considering factors that lead to nonlinearity differ according to their complexity and first of all loads that they take into account.

The most widespread models are those having two freedom degrees in radial directions - this improves results of the rotor system analysis significantly.

Such models are usually built on the Hertz contact theory linking radial loads that act on the bearing and deformation in contact area between the rolling element and the bearing rings. When describing the bearing model it is accepted that there is not any sliding of rolling elements and surfaces. Damping is usually considered in statement of equivalent viscous and linear friction. In calculations with these models the rotor weight, inner bearing clearance are considered, the number of rolling elements in loading area, loading distribution at rolling elements are calculated [6], [7], [8].

The model including defects on the rings and rolling elements develops the one with two freedom degrees [9], [10]. Defects have extent in circular direction and are modeled by changing clearance when the rolling element enters and leaves it.

In the article [4] the two-degree of freedom model of the ball bearing is extended due to possibility to calculate contact stiffness being in the force equation with consideration of the races curvature. These relations are known and given in the multiple reference books on the bearings [11], [12]. Possibility to calculate the main kinematic frequencies of rolling bearings, when geometry and the rotor speed are known, is added. The model is included into the program system Dynamics R4 for analysis of the rotor systems dynamics at unsteady and nonlinear statement considering weight [13]. Analysis of the rotor supported by two nonlinear rolling bearings showed significant influence of the operating modes on the rotor dynamics. There is new possibility to investigate influence of clearance, rotating speed on frequency spectra and amplitude-frequency characteristics for any rotor system supported by rolling bearings.

Taking into account the above, the next step to develop the bearing model is consideration of inertia of rolling elements [14], [15]. Insignificant model complication specifies both frequency and amplitude characteristics of the rotor system.

Angular-contact bearings are loaded with not only radial forces but also axial ones appearing during operation. This force changes relative position of the rings, parallel and axial skew of rotors appear in the bearing clearance. Correspondingly, the bearing stiffness characteristics change.

To consider such multiple-factor loading, the more exact models are required, i.e. five degrees of freedom models described by Harris [16], but developed only lately. The more important achievements in this direction are works [17], [18], [19].

As stated above, creation and use of the models that take into account all factors influencing the bearing will improve the results of quantitative and qualitative analysis of the rotor systems. Considering this, the purpose of the work is to develop nonlinear mathematical models of rolling bearings of two types (the ball and roll ones) and their practical application in the rotordynamic tasks.

The mathematical model that gives accurate enough description of the support with the rolling bearing, ball or roll one, may be included into the model of the multiple shaft rotor system with many supports to carry out the following analysis.

The mathematical models and algorithms of different complexity used to analyze the rotor systems and described in works of Russian and foreign researchers are presented below.

The models of different complexity are presented as force functions where loads appearing in the bearing are the functions of motion parameters of the inner race.

The general motion equation for the rotor system including the rolling bearings may be written in a matrix form as the following:

$$M \cdot \ddot{X} + C \cdot \dot{X} + K \cdot X = F_U + F_B + W \,, \qquad (1)$$

where $X, \dot{X}, \ddot{X}$ – the columns of vibration displacements, speeds and accelerations correspondingly, M – inertia matrix; K – stiffness matrix; C – matrix of damping and gyroscopic forces; F_U – column of unbalanced forces; F_B – column of forces appearing in bearings depending on displacements of the rotor system; W – weight force.

Generally the equation (1) is nonlinear. Loads appearing in bearings of different types are the functions of displacements and speeds, so the exact solution of such equation is possible only at unsteady statement.

Model of bearing with 5 freedom degrees without inertia of rolling elements

The model with 5 freedom degrees was developed by Jones as early as 1960 [20]. Initially there was the plan to use it when calculating under static loading in order to obtain the bearing stiffness, so only axial displacements and inclination of inner race are taken into consideration.

This work gives the algorithm from the article [21]. There are 2 coordinate systems in the article: local and global. Generally displacement of any point along the rotor axis may be described using three linear displacements and two angular ones. Correspondingly, the bearing model should represent the algorithm linking linear and angular rotor displacements with the bearing reactions. The right-handed coordinate systems are used, Figure 1.

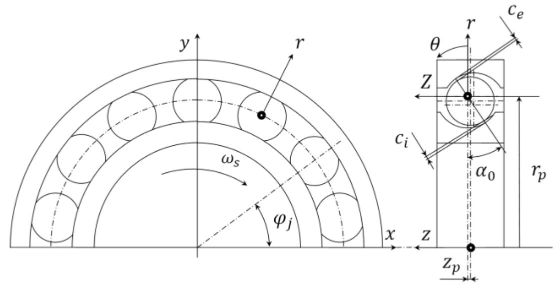

Figure 1 To description of bearing coordinate systems

Model with 5 freedom degrees taking into account inertia of rolling elements

The same basic principles as for the model with 5 degrees of freedom without inertia are used. Considering rolling element inertia, contacts at inner and outer races may take place under different contact angles which is taken into account in the ball equilibrium equations.

Rotor system model

On the basis of the mathematical model described above, the algorithm and program modulus were developed. When displacements and speed of the rotor system are known, this modulus allows obtaining not only force and moment reactions of the bearing as a system element but also its inner parameters – loads distributions at rolling elements at all freedom degrees, contact angles at every rolling element, the ring skew, etc. The program modulus was developed according to ideology and architecture of the Dynamics R4 program system to analyze the rotor dynamics of rotating machines and it was included into the system.

Figure 2 shows the investigated rotor model. The rotor represents a system consisting from two masses simulating the point rotor and the point case. The case is connected with the foundation by elastic links, and point masses are connected with each other by the link simulating angular-contact bearing. Unbalance and axial load influence the rotor.

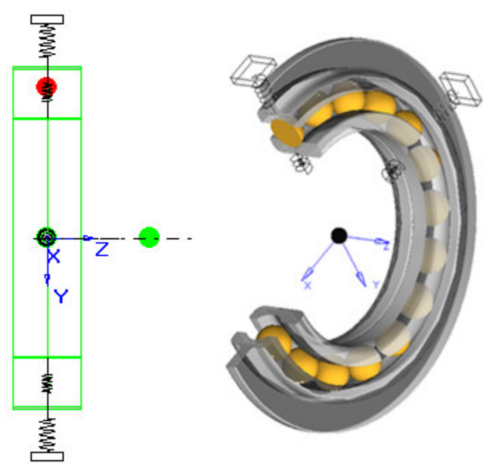

Figure 2 Rotor system model

Table 1 gives initial model data.

Table 1

Parameter	Value
Case mass, kg	10
Suspension stiffness, N/m	2×10^{8}
Suspension damping, N*sec/m	1000
Rotor mass, kg	3
Rotating speed, Hz	3
Rotor unbalance, g*cm	30, 300
Bearing pitch diameter, mm	52
Rolling elements diameter, mm	11,9062
Number of rolling elements	11
Grooves number, mm	6.166
Nominal contact angle, deg	30
Damping in bearing, Nsec/m	2940
Axial force, N	100…1000

Results of static analysis of rotor system

Static analysis of the rotor with the bearing as a part of the rotor system is a difficult task that allows determining the bearing stiffness characteristics. Table 2 shows the matrix of tangent stiffness obtained for the axial load of 100N.

Table 2

	Fx[N]	Fy[N]	Fz[N]	Mx[Nm]	My[Nm]	Mz[Nm]
ux[m]	1.37E+08	-1.19E+02	-1.65E+02	1	8.64E+05	-2.17E+02
uy[m]	-1.19E+02	1.40E+08	2.58E+06	-8.81E+05	-1	0
uz[m]	-1.65E+02	2.58E+06	1.73E+07	-4.38E+04	-2	0
urx[rad]	1	-8.81E+05	-4.38E+04	6.00E+03	0	0
ury[rad]	8.64E+05	-1	-2	0	5.83E+03	-1
urz[rad]	-2.17E+02	0	0	0	-1	0

It should be noticed that only five main coefficients kt00, kt11, kt22, kt33, kt44, kt04 having maximum values may provoke interest. In comparison with these coefficients, the other ones may be neglected in any following analysis of the rotor system – static or dynamic one.

Table 3 shows value of main coefficients of the tangent stiffness matrix depending on axial force acting on the rotor.

Table 3

N	200	400	600	800	1000
Kt00	1.69E+08	2.08E+08	2.34E+08	2.53E+08	2.69E+08
Kt11	1.70E+08	2.09E+08	2.34E+08	2.53E+08	2.69E+08
Kt22	2.39E+07	3.40E+07	4.26E+07	5.00E+07	5.69E+07
Kt33	8.22E+03	1.17E+04	1.46E+04	1.71E+04	1.95E+04
Kt44	8.17E+03	1.17E+04	1.45E+04	1.71E+04	1.95E+04
Kt04	1.12E+06	1.47E+06	1.72E+06	1.93E+06	2.10E+06
Kt13	1.13E+06	1.47E+06	1.72E+06	1.93E+06	2.10E+06

Figure 3 gives curves that correspond to Table 3.

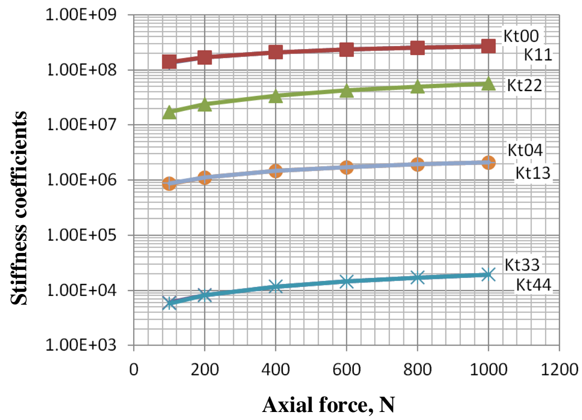

Figure 3 Coefficients of tangent stiffness matrix vs axial force on rotor

Significant influence of axial force, acting on the rotor, on the stiffness bearing characteristics should be noticed. Increase in axial force from 100N to 1000 N leads to increase in stiffness at different coefficients in 1,5-2 times.

It should be noticed that for engineers solving the tasks of general dynamics the problem of stiffness determination of the support unit with the rolling bearing has always been urgent enough. Some coefficients may be found in few sources. However, their values are approximate. So, the table is given in the reference book [1] where flexibility coefficients for ball and roll bearings of different sizes are presented. Table 4 converts their values into stiffness coefficients.

Table 4

Shaft journal diameter, mm	Bearing, N/m	
	ball	roll
30-55	0.125e9 - 0.139e9	0.189e9 - 0.23e9
60-100	0.189e9 - 0.2e9	0.434e9 - 0.4e9

The table gives value of the only coefficient – kt00. In correspondence with these data, the value for the investigated bearing accounts for 0.189e9 - 0.2e09 N/m.

Here kt00 values are in range from 0.169e9 to 0.269e9 N/m. Unfortunately, quite close values may be obtained only for this bearing dimension size. Possible divergence may achieve several hundred percent for other dimension sizes.

It should be noticed that nowadays static task on determination of stiffness bearing characteristics may be solved at nonlinear contact statement by the finite element method. For example, it may be solved using the widely-used ANSYS system. However, manhours that are required to solve this task are significantly higher than those obtained according to the given methodology. To solve the rotor dynamic tasks, tangent stiffness matrixes of all rotor system bearings should be preliminary obtained for different axial forces acting on them.

Let us consider the task on determination of radial force influence on the stiffness bearing characteristics. It should be noticed that the rotor weight may achieve several tens kilograms even for such dimension of the rolling bearing. Figure 4 shows the obtained results for several values of static force applied to the bearing along weight force at constant axial force of 100 N.

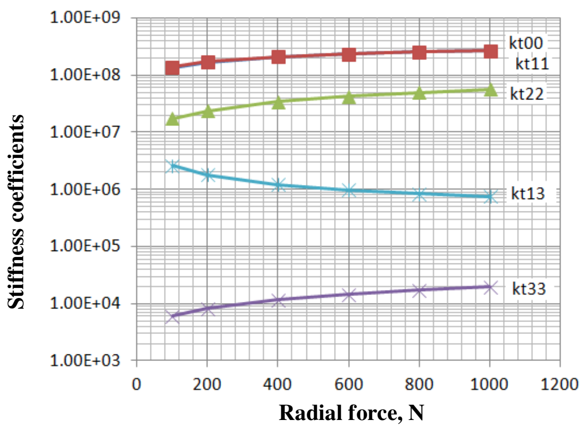

Figure 4 Coefficients of tangent stiffness matrix vs radial force on rotor

Figure shows that increase in radial force leads to change in coefficients of tangent stiffness matrix. When increasing axial force, radial force impact on stiffness coefficients lowers rapidly.

Unbalance force influences the bearing stiffness like the weight force. It is obvious that this influence may be neglected for heavy and well-balanced rotors where unbalanced force at maximum regimes does not exceed 10…20% from weight force.

Tangent stiffness matrixes may be efficiently used in linear analysis of the rotor system for the highlighted regime with constant rotating speed, where axial and unbalanced force does not change. Similar linearization of nonlinear systems in case of complex dynamic systems may take less time for their analysis keeping calculation accuracy acceptable in practice.

Results of dynamic analysis of rotor system

Dynamic analysis of the rotor system was carried out for axial force 100 N and unbalance 30 gcm in the range up to 50000 rpm.

To solve the task, the equation (1) is integrated directly using the adaptive integration algorithm. Integration step is varied automatically during calculation in correspondence with the given accuracy and current gradients of the simulated process. The obtained amplitude-frequency characteristic is the same both for Y and X direction. It is presented in Figure 5.

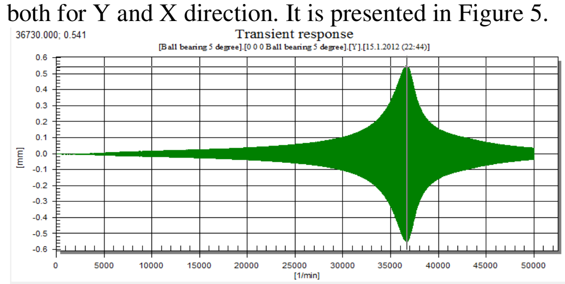

Figure 5 Amplitude-frequency characteristic of rotor with non-linear bearing

The following pictures show how the bearing stiffness changes depending on rotating speed, Figure 6…12.

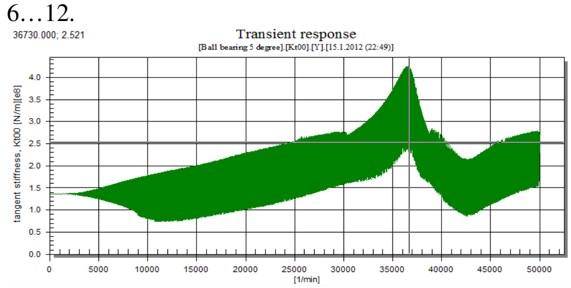

Figure 6 Change in kt00 coefficient of tangent stiffness matrix

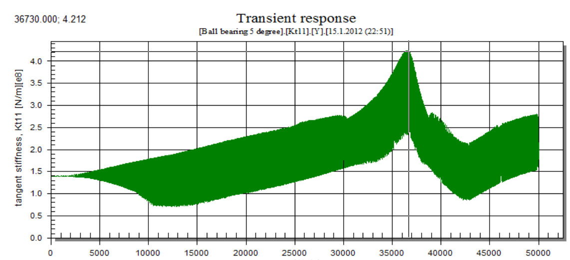

Figure 7 Change in kt11 coefficient of tangent stiffness matrix

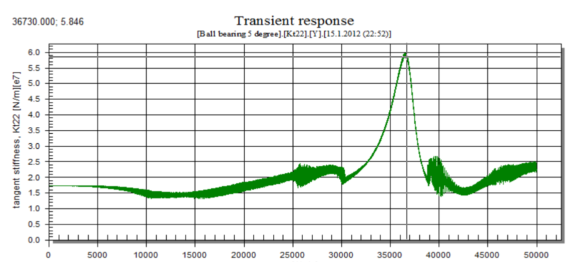

Figure 8 Change in kt22 coefficient of tangent stiffness matrix

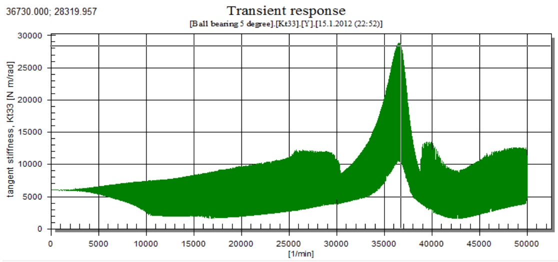

Figure 9 Change in kt33 coefficient of tangent stiffness matrix

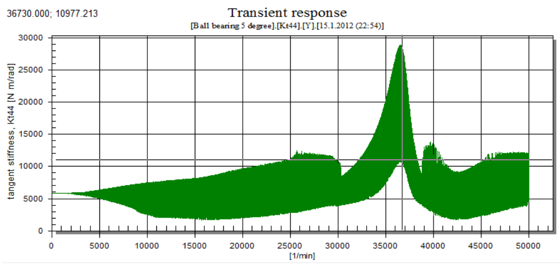

Figure 10 Change in kt44 coefficient of tangent stiffness matrix

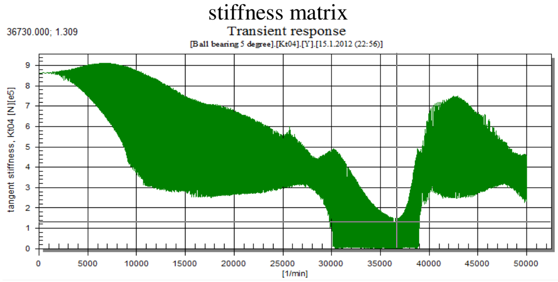

Figure 11 Change in kt04 coefficient of tangent stiffness matrix

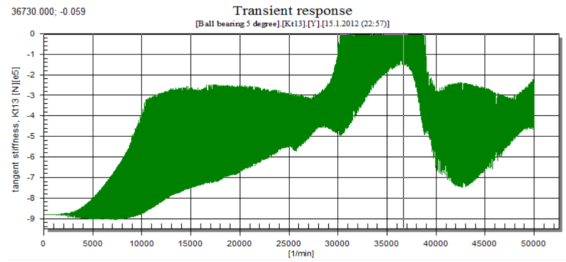

Figure 12 Change in kt13 coefficient of tangent stiffness matrix

These pictures show that coefficients of stiffness matrix may significantly change both for one loading cycle and in the whole calculated range. Maximum change corresponds to resonance regime. Meanwhile, stiffness coefficients may change in 3 and more times.

Conclusion

The mathematical models, algorithms and program modulus to analyze rotors, supported by nonlinear angular-contact rolling bearings, were developed. The program modulus is included into the Dynamics R4 program system for analysis of vibration characteristics of the complex rotor systems. It is proved that stiffness bearing characteristics depend significantly on static and dynamic loads acting on it and unbalanced forces. Analysis of the rotors supported by rolling bearings at nonlinear statement may specify the obtained results significantly.

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