MODELING AND ANALYSIS OF DYNAMIC CHARACTERISTICS OF TURBOPUMPS WITH ANNULAR SEALS

Davydov A.V., Degtiarev S.A., Ivanov A.V., Leontiev M.K.

Information about authors

Davydov Arkadiy V., post-graduate student of Moscow Aviation Institute (National Research University) MAI, Volokolamskoe shosse, 4, Moscow, A-80, GSP-3, 125993; tel.: +7-906-783-91-68; e-mail: davidovarc@gmail.com.

Degtiarev Sergei A., course leader of the scientific and technical centre of rotordynamics Alfa-Tranzit Ltd., Russia, Moscow region, 141400, Khimky, Leningradskaya street, 1; tel: +7 -916- 516-75-27, e-mail: dегs@alftran.com.

Ivanov Andrei V., associate professor of rocket engines department of Voronezh State Technical University; candidate of technical science. VSTU, 394006, Russia, Voronezh, Moskovskij av., 14. Tel: 8-473-262-97-16, e-mail: jav308@inbox.ru.

Leontiev Mikhail K., professor of Moscow Aviation Institute (National Research University), Doctor of Technical Science, MAI, Volokolamskoe road., 4, Moscow, A-80, GSP-3, 125993; tel.: +7-985-768-71-29; e-mail: lemк@alftran.com.

Abstract

The general methodology of analysis of turbopumps with annular seals at linear and non-linear statement is presented. For annular seals with floating rings the algorithm allowing consideration of hydrodynamic forces, rings inertia and friction forces is developed. The results of analysis of the turbopump rotor system with annular seals and their influence on vibration characteristics of the rotor system are presented.

Key words: turbopump, rotordynamics, nonlinear models, annular seals, floating rings, DYNAMICS R4.

Introduction

Subject of study is a hydrogen turbopump supported by four rolling bearings and having three gas annular seals and five fluid ones of different types. The turbopump was developed in OSC KBKhA [1]. Figure 1 gives the general view of the turbopump rotor supported by rolling bearings.

Figure 1. General view of hydrogen turbopump rotor

The rotor is on two supports. Duplex radial single-
row bearings are used in the supports. The annular plate elastic-damping supports of dry friction are placed between outer bearings rings and the turbopump case. Leaktightness of air-gas channel of the turbine and the impeller is provided by annular seals of different types. In one of designs of the turbine stage turbopump, the shaft and the impellers are made from a titanium alloy. The main turbopump peculiarities are its small dimensions and high rotating speed.

From view of general rotor dynamics, annular seals in turbopumps act as additional supports influencing the position of resonance regimes and their amplitude [2, 3]. Besides that, hydrodynamic forces appearing in annular seals during operation may lead to the rotor autorotation and its loss in stability, becoming apparent as big reactions in supports. So far, for designers obtainment of seals’ dynamic characteristics has been still quite laborious task to solve. The aim of the present work is creation of practical methodology and computational algorithm allowing to estimate influence of an annular seal with a floating ring on the general rotor dynamics, and their application for analysis of vibration characteristics of the simulated turbopump. Investigations were hold in the Dynamics R4 (www.alfatran.com) program system for solution of rotordynamics tasks of turbomachines and units for different purposes.

**Model of rotor system**

To describe the rotor geometry, finite elements including into the program library were used. Those elements are: rod beam elements, inertia elements, elastic and rigid links, etc. Every rotor detail was simulated separately and included into the general rotor model as an independent subsystem. Rigid or elastic links, simulating junctions and fits in the rotor, and its supports units, were set between subsystems. Residual rotor unbalance was given. The rotor bearings were loaded by axial forces. The turbopump case in this investigation was simulated by the rigid beam connected with the case by two rigid links. Figure 2 gives the general turbopump dynamic model.

Table 1 presents the main parameters.

![Figure 2. Model of turbopump rotor](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor length, mm</td>
<td>275</td>
</tr>
<tr>
<td>Rotor mass, kg</td>
<td>1.66</td>
</tr>
<tr>
<td>Polar inertia moment, kg·m²</td>
<td>0.00076</td>
</tr>
<tr>
<td>Axial force on bearings, N</td>
<td>200</td>
</tr>
</tbody>
</table>

Value of stiffness and damping coefficients of elastic-damping support 1 and elastic-damping support 2 are accepted to be constant and shown in Table 2.

<table>
<thead>
<tr>
<th>Support</th>
<th>Coefficient of radial stiffness, N/m</th>
<th>Damping coefficient, N·sec/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic-damping support 1 (turbine support)</td>
<td>8.6·10⁶</td>
<td>500</td>
</tr>
<tr>
<td>Elastic-damping support 2 (pump support)</td>
<td>1.5·10⁷</td>
<td>500</td>
</tr>
</tbody>
</table>
The first three seals (S1, S2 and S3) are gas, and are not included into the investigated rotor model, because their influence on the rotor dynamics is insignificant comparing with fluid annular seals. The main parameters of annular hydraulic seals in the turbopump model are averaged and presented in Table 3. Pressure difference is given at maximum regime and it changes parabolically with rotating speed change.

Table 3

<table>
<thead>
<tr>
<th>Seal</th>
<th>Floating ring</th>
<th>Floating ring</th>
<th>Floating ring</th>
<th>Annular without ring</th>
<th>Floating ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notations</td>
<td>S4</td>
<td>S5</td>
<td>S6</td>
<td>S7</td>
<td>S8</td>
</tr>
<tr>
<td>Length, mm</td>
<td>7.5</td>
<td>6.8</td>
<td>7.5</td>
<td>5</td>
<td>6.8</td>
</tr>
<tr>
<td>Diameter, mm</td>
<td>22</td>
<td>49</td>
<td>22</td>
<td>85</td>
<td>49</td>
</tr>
<tr>
<td>Clearance, mm</td>
<td>0.0443</td>
<td>0.005</td>
<td>0.0271</td>
<td>0.077</td>
<td>0.006</td>
</tr>
<tr>
<td>Pressure difference, Pa</td>
<td>3.5e6</td>
<td>8.29e6</td>
<td>13.8e6</td>
<td>3.91e6</td>
<td>6.48e6</td>
</tr>
</tbody>
</table>

Linear analysis of turbopump rotor

The most general rotor analysis is stationary, where critical speeds of the linear rotor models, mode shapes are obtained and amplitude-frequency characteristics from unbalance are calculated.

The task has solution at linear statement, i.e. for every regime the system may be considered as linear: and natural frequencies, relative damping coefficients, energy distribution on shapes, etc. may be obtained. The rotor amplitude frequency characteristics may be built as a function of unbalanced forces. It should be noticed that linearised statement is used for ordinary annular seals or for the case when friction forces stop the seal ring . In other cases analysis at nonlinear statement is required to carry out.

Results of analysis of the rotor supported by angular contact bearings without fluid seals and with them are presented below. In first case the rotor analysis may be hold for the turbopump rotor investigation and data obtainment for its balancing. These results may be used for the rotor model validation according to the amplitude frequency characteristics obtained while balancing. The second case corresponds to the rotor with fluid seals, i.e. the turbopump characteristics are considered at its work.

Estimation of stiffness of the turbopump rotor bearings showed that for the given bearing dimension type all the stiffness coefficients depend on the regime insignificantly and may be accepted as constant. As an example, Table 4 gives symmetric matrix of stiffness coefficients of the turbine bearing of 5*5 dimension. Similar results are obtained for the other bearings.

Table 4

\[
\begin{array}{cccc}
K_{11} &=& 7.07E+07 & K_{22} = 7.07E+07 \\
K_{12} &=& 7.40E+02 & K_{23} = 1.51E+06 \\
K_{13} &=& 4.00E+04 & K_{24} = 4.71E+05 \\
K_{14} &=& 0 & K_{25} = 0 \\
K_{15} &=& -4.71E+05 & K_{33} = 3.31E+03 \\
K_{25} &=& 0 & K_{34} = 1.22E+04 \\
K_{35} &=& -3.33E+02 & K_{44} = 3.31E+03 \\
K_{45} &=& 0 & K_{55} = 3.31E+03
\end{array}
\]

Stiffness coefficients \( K_{xx} \), \( K_{xy} \) and damping coefficients \( C_{xx} \), \( C_{xy} \) for the ordinary annular seal is a function of the seal geometry, the rotor speed, fluid characteristics, pressure difference are obtained for the
central position of the rotor shaft in clearance. They are calculated according to the methodology developed by Childs and presented in the work [4]. Their values for the investigated turbopump are obtained in Dynamics R4, and are shown in Figure 3 for one of the hydraulic seals.

Figure 3. Stiffness coefficients of S4 seal

Influence of annular seals on the rotor dynamics is presented below. Table 5 presents damped rotor critical speeds and the corresponding mode shapes with forward precession. Values of relative damping coefficient are shown in brackets. Mode shapes of backward precession and axial oscillations are excluded.

Table 5

<table>
<thead>
<tr>
<th>Rotor without annular seals</th>
<th>Rotor with annular seals</th>
</tr>
</thead>
<tbody>
<tr>
<td>24964.1 (5.25 %)</td>
<td>25666.8 (8.91 %)</td>
</tr>
<tr>
<td>44625.8 (4.65 %)</td>
<td>49252.0 (28.47 %)</td>
</tr>
<tr>
<td>97316.6 (4.39 %)</td>
<td>111182.1 (34.51 %)</td>
</tr>
</tbody>
</table>

Comparison of results shows that annular seals toughen the rotor and move resonance regimes in the area of high frequencies. Mode shapes of the rotor with annular seals are overdamped and have spatial characteristic. Figures 5, 6 show the plot of loads on the supports in places of elastic-damping packages position (unbalance 1-0.11 gcm, unbalance 2 – 0.01 gcm) vs. rotating speed.

Cross-sectional stiffness and damping coefficients have the same value but unlike sign which indicates presence of circulating forces in clearance that may lead to the rotor loss in stability.
The obtained results show significant seals influence on the turbopump rotor. Position and amplitude of resonance regimes change.

Nonlinear analysis of turbopump rotor

The seal reaction on the dynamic system is determined by hydrodynamic force appearing in the seal slot. In linear analysis the ring is fixed, and the seal is an ordinary annular one. The general rotor motion equation may be written as following in the matrix form

\[ [M]\ddot{q}(t) + [C]\dot{q}(t) + [K]q(t) = (F(t))_0 + (R^{seal}(t)), \]

where \([M],[C],[K]\) – matrix of inertia, damping and gyroscopic forces, and stiffness correspondingly, obtained for the finite element model of the rotor; \([\ddot{q}],[\dot{q}],[q]\) - vectors-columns of generalized accelerations, velocities and displacements correspondingly; \((F(t))_0\) – vector-column of unbalance forces; \((R^{seal})\) – vector-column of seal reaction.

As projection on X and Y axes these reactions may be written in the following way

\[
\begin{bmatrix}
R_x^{seal} \\
R_y^{seal}
\end{bmatrix} = -\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix} \begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix} - \begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix} \begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix}.
\]
In this analysis it is supposed that the ring has inertia and can move, i.e. the rotor system becomes nonlinear. Then solution of the motion equation is found by direct integration of seal’s reaction.

Let us consider the algorithm of determination of the seal’s reaction on the shaft rotor movement. At initial moment of time the sealing ring is considered as fixed and centered, i.e. the seal works as an ordinary annular one. Conditions of the ring fixity are the following:

\[ |\vec{F}_g^{ring}| < F_{\mu}^{rest} |\vec{F}_g^{ring}| < F_{\mu}^{sliding}, |\vec{v}_n^{ring}| < v_{\min}, \]

where \( \vec{F}_g = -\vec{F}_g = -\{F_{gx}, F_{gy}\} \) – hydrodynamic force vector; \( F_{\mu}^{rest} \) and \( F_{\mu}^{sliding} \) – static and sliding friction force correspondingly; \( \vec{v}_n^{ring} = \{v_{x}^{ring}, v_{y}^{ring}\} \) – ring velocity vector; \( v_{\min} \) – minimum sliding speed.

The ring starts moving when hydrodynamic force overcomes static friction force \( F_{\mu}^{rest} \), considering that \( F_{\mu}^{rest} \geq F_{\mu}^{sliding} \). Vector of static friction force \( F_{\mu}^{rest} \) at the moment of movement start is opposite to the hydrodynamic force vector \( \vec{F}_g \). In this case the ring gets acceleration and moves under inertia force:

\[ \vec{F}_\omega = a_n^{ring} m_n^{ring} = (\vec{F}_{g}^{ring} + F_{\mu}^{rest}), \]

where \( m_n^{ring} \) - ring mass; \( \vec{F}_\omega \) - ring inertia force.

If sliding and static friction coefficients are different, the following motion takes place with consideration of sliding friction force which is opposite to the vector of the ring speed \( \vec{v}_n^{ring} \):

\[ \vec{F}_\omega = a_n^{ring} m_n^{ring} = (\vec{F}_{g}^{ring} + F_{\mu}^{sliding}). \]

Absolute values of sliding and static friction forces change with change in differential pressure and they are calculated as

\[ F_{\mu}^{rest} = \mu_{rest} \cdot F_r, \]

\[ F_{\mu}^{sliding} = \mu_{sliding} \cdot F_r, \]

\[ F_r = (P_1 \cdot S_{h1} - P_2 \cdot S_{h2} + F_s), \]

where \( \mu_{rest} \) and \( \mu_{sliding} \) – static and sliding friction coefficients correspondingly; \( S_{h1} \) and \( S_{h2} \) – areas corresponding to sizes of the \( h_1 \) and \( h_2 \) rings. Influence of change in areas \( S_{h1} \) and \( S_{h2} \) on friction forces during the ring motion is not considered.

Reactions on the seal rotor and the case with the floating ring are different. In case when the ring is fixed, reaction on the shaft is equal to hydrodynamic force, and reaction on the case is opposite to it:

\[ -\vec{R}_{case} = \vec{R}_{shaft} = \vec{F}_g. \]

In the ring moves, only reaction from the friction force is transmitted on the case:

\[ -\vec{R}_{case} = \vec{F}_g^{sliding}, \]

on the rotor – reaction from hydrodynamic force calculated with consideration of the ring inertia:

\[ \vec{R}_{shaft} = \vec{F}_g. \]

Considering the shaft and the ring displacements from the central position and speeds of their motion, the value of hydrodynamic force as projections on the X and Y axes may be obtained through stiffness coefficients \( C_{xx}, C_{xy} \), found for the central rotor position in clearance.

\[ F_{gx} = -(K_{xx} \cdot \Delta u_x + K_{xy} \cdot \Delta u_y + C_{xx} \cdot \Delta v_x + C_{xy} \cdot \Delta v_y); \]
\[ F_{gy} = -\left( K_{yx} \Delta u_x + K_{yy} \Delta u_y + C_{yx} \Delta v_x + C_{yy} \Delta v_y \right) \]

where:

\[ \Delta u_x = u_x^{shaft} - u_x^{ring}; \quad \Delta u_y = u_y^{shaft} - u_y^{ring}; \]

\[ \Delta v_x = v_x^{shaft} - v_x^{ring}; \quad \Delta v_y = v_y^{shaft} - v_y^{ring}. \]

Operation of the algorithm on determination of the ring reaction may be presented as a flow chart, Figure 8. At initial time moment the ring has a central position. Input parameters for the ring reaction calculation at \( n \) integration step are \( u_x^{shaft}, u_y^{shaft}, v_x^{shaft}, v_y^{shaft}, \Delta t_n. \)

Figure 9. Flow chart of algorithm for determination of seal reaction

Figure 10 shows the vector schemes of the ring motion at acceleration and deceleration. At acceleration the rotor and the ring centers move round a circle with eccentricities \( e^{rotor} \) and \( e^{ring} \) correspondingly, during motion they increase. Figure 10 a) shows that sum of hydrodynamic force vectors \( F^{ring} = -F \) and sliding friction force \( F^{sliding} \), acting on the ring centre, is equal to inertia force \( F_\omega \). It causes acceleration \( \alpha^{ring} \), whose tangent component is \( \alpha^{ring} \) increases the ring motion speed \( \omega^{ring}. \)

a)  

b)
When hydrodynamic force starts decreasing, the ring motion speed reduces. The main differences of the ring deceleration from its acceleration are that the rotor eccentricity is lower than the ring eccentricity and continue decreasing, so tangential component of acceleration is aimed at speed decrease. During both the ring deceleration and acceleration, hydrodynamic force vector may be bigger than friction force vector, but because of the phase difference between them acceleration works whether to the ring acceleration or its speed-up.

Figure 11 shows the amplitude-time characteristics of the rotor in the section of elastic-damping support 1, obtained by direct integrating of the rotor motion equations at small unbalances.

Figures 12 and 13 show change in friction forces of the ring and hydrodynamic force in seals S5 and S8. In the range up to 140000 rpm hydrodynamic force does not exceed friction force, i.e. rings do not move. Similar results are obtained for the other seals.
Corresponding to these characteristics forces acting in seals S4, S5, S6, S7 and S8, are shown in Figures 15, 16, 17, 18 and 19.

**Figure 15** Forces acting on seal ring S4

**Figure 16** Forces acting on seal ring S5

**Figure 17** Forces acting on seal ring S6

**Figure 18** Hydrodynamic force in annular seal S7 (without ring)

**Figure 19** Forces acting on seal ring S8

Figures show that for seals S4, S5 and S6 friction force is more than hydrodynamic force, and rings do not move (speeds curves are absent). In case of S8 seal, when hydrodynamic force exceeds the ring friction force, its motion appears which continues up to ~ 110000 rpm.

**Conclusion**

The present work gives the methodology and algorithms of nonlinear analysis of high speed turbopump rotors with annular seals and specifically seals with floating rings. Algorithms take friction forces, inertia ring characteristics into account and allow following their motion during analysis and influence on the rotor
dynamics. In case of small differential pressure, weak axial prepressure of the rings by springs or big unbalances, hydrodynamic force appearing in the seal may exceed friction force and rings may start moving, changing position of the rotor resonance regimes and their amplitude. Using these data, a designer can make a decision on necessity to change the ring construction and choose its operating mode.

**Literature**


2. Иванов А.В. Исследование влияния уплотнений на динамические характеристики высокооборотного ротора. Вестник СГАТУ им. Королева. №3 (34) Часть 1. 2012 г. с.303-308
