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DYNAMICS OF UNBALANCED ROTOR SUPPORTED BY ROLL BEARING

Nonlinear effects in rolling bearings are related to contact deformation of rolling elements, clearances between rolling elements and the bearing races. The rotor motion with such bearings is quite complicated– from ordered to chaotic. The article presents the roll bearing model taking into account contact stiffness, clearances between rolling elements and races, external loads in correspondence with Hertz theory. Using it, behaviour of unbalanced rotor system with nonlinear bearing was investigated.

Key words: *roll bearing model, clearance, stiffness, rotor dynamics, Dynamics R4, transient response.*

Introduction

Nowadays transient response of rotor systems is one of the main tasks which go from just academic statements and premises into practice in work of many engineers. Work in this direction has been carried out for a long time. Works of Kelzon A.S. should be noted among Russian authors [1]. One of the fundamental works in numerical analysis of rolling bearings that became the basic one for the work of many foreign authors is the work of Harris [2]. Works of Fleming [3], Randall [4], who investigated rotor dynamics with two-degrees of freedom bearing, and many others are still working in this direction. The important landmark in the bearing models development that may be applied in rotor dynamics, is work of De Mul who presented the bearing with five freedom degrees [5]. The present article gives some results of investigation of dynamic behaviour of the rotor systems supported by rolling bearings and specifically by radial roll bearings.

General theory

Generalized motion equation for the rotor system including the rolling bearings may be written in the following way:

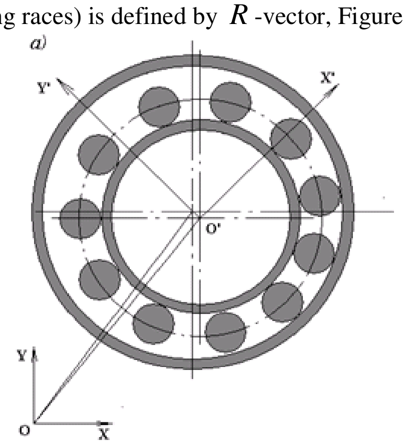
$$M \cdot \ddot{X} + C \cdot \dot{X} + K \cdot X = F_U + F_B + W,$$

where M – inertia matrix of the rotor system; C – matrix of damping and gyroscopic forces; $\ddot{X}, \dot{X}, X$ – columns of vibration acceleration, speed and displacement correspondingly; F_U – column of unbalanced forces; F_B – column of forces appearing in bearings and depending on displacements and speeds of the rotor system; W – weight force. In general case forces appearing in bearings are the functions of displacements and velocities, so the exact solution of such equation is possible only at unstable statement by direct integrating of the motion equation.

Bearing model

As a part of the rotor system, the radial roll bearing with cylindrical rolling elements bears and transmits only radial loads from the rotor to the stator. Load is transmitted through several rolling elements being in the loading zone. The number of rolling elements taking part into loading transmission is determined by the clearance value in the bearing, contact stiffness and characteristics of the rotor system – the rotor weight, rotating speed, and the rotor eccentricity in the bearing clearance. There are quite simple models which allow calculating of elastic characteristics of the rotor support including the rolling bearing depending on the operating mode of the rotor system. They are mainly presented by the dynamic system with two freedom degrees and use the well-known Hertz theory.

Let us give the main equations of this theory concerning the roll bearing [4], [5]. In the model description the following assumptions are accepted – inertia of rolling elements is not taken into account, there is only linear viscous damping in the bearing, and there is no any sliding of the rolling elements. Figure 1 shows the model of the roll bearing. At every time moment the relative position of the rotor and stator (inner and outer bearing races) is defined by R -vector, Figure 1.



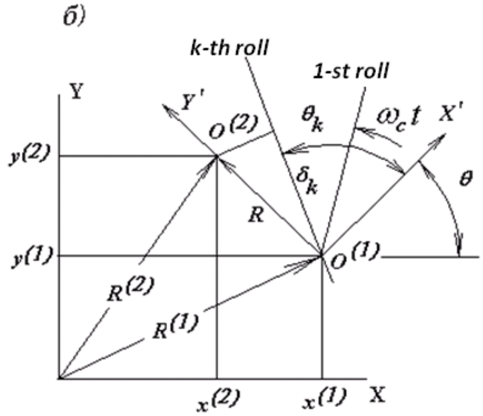


Figure 1 – Model of roll bearing
a) schematic image of roll bearing; б) relative position of inner and outer bearing races

Vector R of the relative displacement of the centers of the bearing races may be found as

$$R = \sqrt{x^2 + y^2}\ ,\ x = x^{(1)} - x^{(2)},$$

$$y = y^{(1)} - y^{(2)},$$

where $x^{(1)}, y^{(1)}$ - projections of vector $R^{(1)}$, determining the position of the center of the bearing inner race; $x^{(2)}, y^{(2)}$ - projections of vector $R^{(2)}$, determining the position of the center of the bearing outer race.

Position of the k-th roll (k=1,2… N) is defined by θ_k angle:

$$\theta_k = \omega_c \cdot t + (k-1) \cdot \frac{2 \cdot \pi}{N},$$

where ω_c - rotating speed of the cage that is calculated using the following equation

$$\omega_c = \frac{\omega^{(1)}}{2} \cdot \left(1 - \frac{D_r}{D_i + D_r}\right) + \frac{\omega^{(2)}}{2} \cdot \left(1 + \frac{D_r}{D_i + D_r}\right)$$

$\omega^{(1)}, \omega^{(2)}$ - in general case rotating speed of the inner and outer race correspondingly (for the stator $\omega^{(2)} = 0$), D_r - roll diameter, D_i - diameter of the inner race, N - the rolls number.

Relative distance between the inner and outer races in direction of θ_k angle is found as

$$\delta_k = R \cdot Sin\theta_k .$$

Let us designate the radial clearance in the bearing as δ. In accordance with the Hertz theory [2] contact stiffness K_H is calculated for linear contact, then radial force for k-th roll will be written as

$$f_k = K_H \cdot (\delta_k - \delta)^{10/9} \quad if \quad \delta_\kappa \geq \delta ;$$

$$f_k = 0 \quad if \quad \delta_\kappa < \delta .$$

Projections of overall reaction of the rolls being in contact

$$F_{X'} = -\sum_{k=1}^{N} f_k \cdot Cos\theta_k\ ,\ F_{Y'} = -\sum_{k=1}^{N} f_k \cdot Sin\theta_k .$$

Transferring back reactions into the overall coordinate system, we get the following

$$F_X = F_{Y'} \cdot Cos\theta + F_{X'} \cdot Sin\theta ,$$

$$F_Y = F_{Y'} \cdot Sin\theta + F_{X'} \cdot Cos\theta ,$$

$$Cos\theta = x / R, \quad Sin\theta = y / R .$$

On the basis of the presented mathematical model of the nonlinear roll bearing, algorithm and the program module was developed as a part of the program system for calculation and analysis of the vibration characteristics of turbomachines DYNAMICS R4 [6], which allows solving tasks of the nonlinear rotor dynamics at unstationary statement.

Model of rotor system

To solve the task, the rotor system model, including the point case with the support, the point rotor and the bearing, is used, Figure 2.

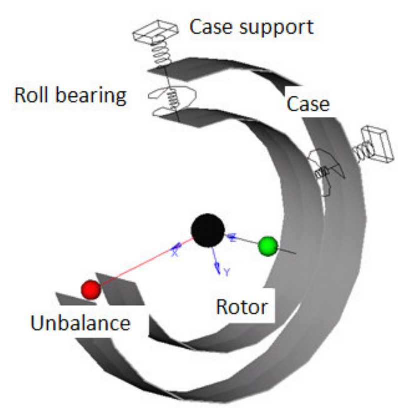


Figure 2 – Model of rotor system with roll bearing in program system DYNAMICS R4

Table 1 shows the parameters of the rotor system model.

Table 1 – Parameters of rotor system

Roll bearing SKF NJ 205 ECML	
Outer diameter,mm	52
Inner diameter, mm	25
Bearing races width, mm	15
Diameter of inner race, mm	31.5
Rolls diameter, mm	8
Rolls number	8
Damping coefficient, Nsec/m	100
Contact stiffness, N/m	$1x10^8$
Point case	

Mass, kg	10
Stiffness of case support, N/m	0.1e9
Point rotor	
Mass, kg	3
Inertia moments, kgm^2	0.001

The system has 5 freedom degrees: the rotor - 3, the case – 2 (the case does not rotate). The rotor is modeled by the point inertia element, the case – by point mass fixed on its support. Weight and unbalanced forces act on the rotor. There is the roll bearing between the rotor and the stator. Damping in the bearing is described by the model of viscous friction. Contact stiffness between rolling elements accounts for $1x10^8$ N/m.

Transient response of rotor system

Figure 3 shows the amplitude-time characteristic of the investigated model with zero clearance in the bearing. Integration of the motion equations is carried out in the frequency range from 0 up to 30000 rpm. Integration time is 10 seconds.

Oscillation beats after transition via resonance are plainly seen in the graph, that relates to the natural vibrations of the rotor system.

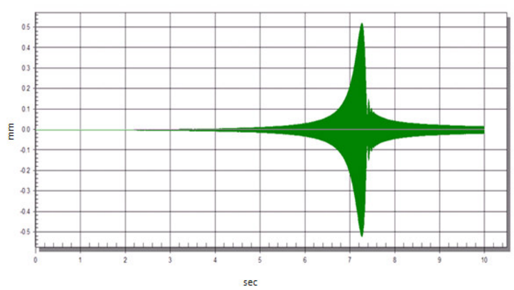


Figure 3 – Amplitude-time characteristic of rotor system
(clearance in bearing is equal to 0.0 mm, unbalance is 10 g*cm)

Because of increase in loads acting on the bearing when approaching to resonance, elastic deformations of loaded rolls and the bearings races take place. Meanwhile, if clearance is small, the number of loaded rolls may increase. The nonlinearity of the bearing is caused by the number of rolling elements in the loading zone - this also changes the bearing stiffness, Figure 4.

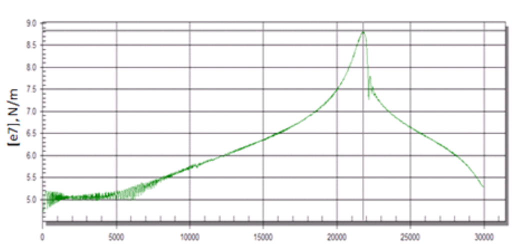


Figure 4 –Radial stiffness of bearing as a part of rotor system model vs regime (clearance in bearing is equal to 0.0 mm, unbalance - 10 gcm)

Almost double change in the coefficient of the bearing stiffness in the investigated frequencies range may be noticed. Meanwhile, the maximum bearing stiffness corresponds to the resonance of the rotor system. Figure 5 shows the waterfall diagram of vibration spectra of the rotor system. In the presented scale of the graph only the rotor harmonic may be highlighted.

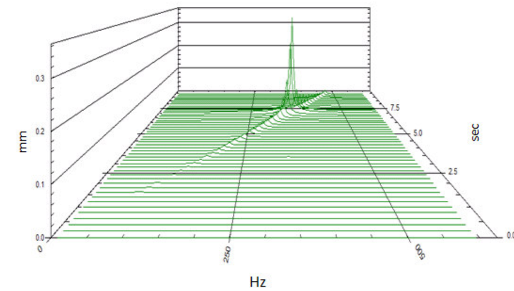


Figure 5 – Waterfall diagram of vibration spectra of rotor system (clearance in bearing is equal to 0.0 mm, unbalance - 10 g*cm)

Let us note the possible errors in resonance frequency position which may take place using the linear model of the rotor system with constant stiffness coefficients in relation to the results for the rotor with the nonlinear bearing (resonance frequency is equal to 21803 rpm), Figure 6. Stiffness coefficient of the linear analog of the bearing is in the range from 0.5e8 N/m до 0.9e8 N/m.

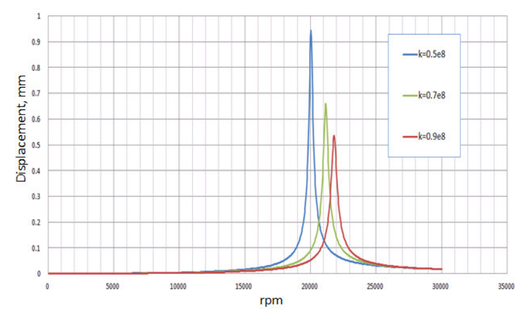


Figure 6 – Amplitude-frequency characteristic of rotor system at different stiffness coefficients (clearance is equal to 0 mm, unbalance is equal to 10 g*cm)

Table 2 shows the errors obtained at different stiffness coefficients in relation to resonance frequency calculated in transient response.

Table 2- Errors on determination of resonance frequency in linear model

Stiffness coefficient, N/m	Resonance frequency, rpm	Amplitude, mm	Error on frequency, %	Error on amplitude, %
0.5e8	20040	0.943	8.1	80
0.7e8	21150	0.659	3.0	26
0.9e8	21810	0.523	~0	~0

Error on frequency is within practical accuracy of the task, but error on amplitude is quite significant.

Rotor model with clearance in bearing

It should be noticed that clearances influence in bearings is determining for the rotor systems behaviour. Figure 7 shows amplitude-time characteristic of the rotor with clearance in the bearing of 0.08 mm. The rotor speeds up to 30000 rpm (50 seconds of integrating) and then stops (50 seconds of integrating more). Table 3 gives the motion orbits.

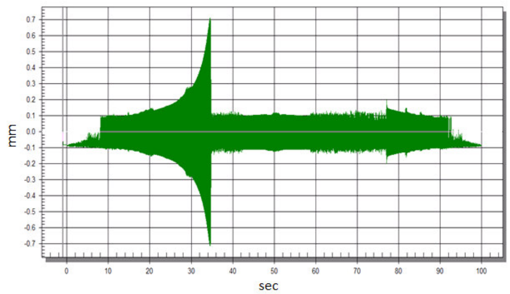
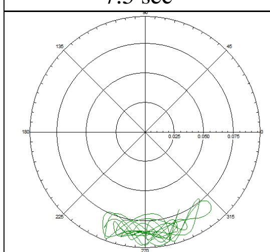
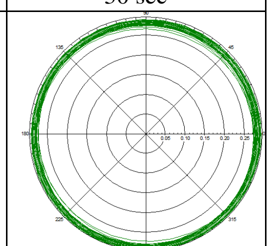


Figure 7 – Amplitude-time characteristic of rotor system
(clearance in bearing is equal to 0.08 mm, unbalance - 10 g*cm)

Table 3 – Orbits of rotor motion

7.5 sec	30 sec
	
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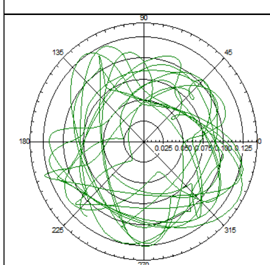
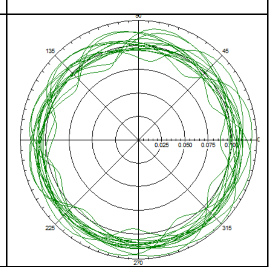
	

Transfer of time signal into frequency area allows obtaining waterfall diagram of vibration spectra, Figure 8.

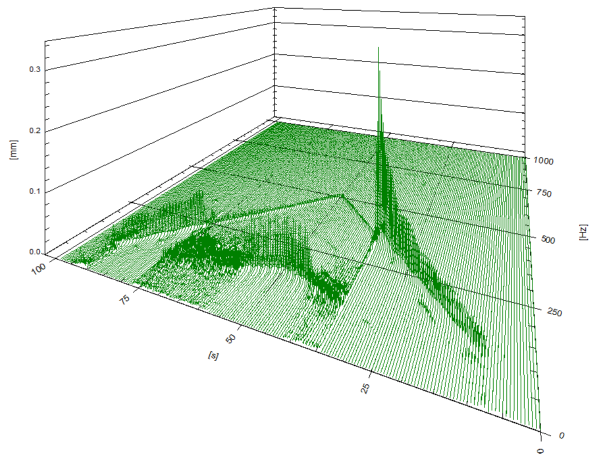


Figure 8 – Waterfall diagram of vibration spectra of rotor system
(clearance in bearing is equal to 0.08 mm, unbalance is 10 g*cm)

When analyzing the obtained motion, the following parts may be seen:

- the part of chaotic motion at the initial stage of the rotor motion up to the second integration second (the rotor is on the bearing race);
- the part where the rotor makes stable motion that is close to the circular one under unbalanced force;
- the part where resonance is passed, and the subsequent rotor behaviour is accompanied by insignificant forced oscillations caused by unbalance and chaotic motion turning into parametric oscillations [7].

The described analysis is the classical one for nonlinear rotor dynamics and clearance influence on the unbalanced rotor behaviour. Figure 9 shows mean value of the obtained rotor motion at speedup and rundown. The curve has bifurcation zone leading to the rotor failure and its chaotic motion. The resonance position at the rotor speedup is determined by the corresponding bearing stiffness. Amplitude-time characteristic of the rotor rundown differs from that one of speedup, is not accompanied by resonance and is characterized by chaotic motion and parametric oscillations up to 14000 rpm.

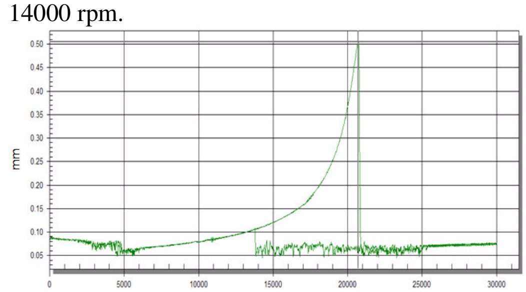


Figure 9 – Mean value of amplitude-time characteristic of rotor system (clearance in bearings is equal to 0.08 mm, unbalance - 10 g*cm). Speedup is up to 30000 rpm and rundown up to 0 rpm

Chaotic rotor motion

The term "chaotic" is used for such motions whose trajectories depend on initial conditions significantly [8]. For the stable rotor work (absence of chaotic motion) the rolling bearings should be loaded by some minimal load. Such load provides the bearing with stable operation, without the rolls sliding on the races. There are certain requirements to its value. For instance, the SKF company obtains the required minimal load of the single-row cylindrical roll bearing using the following equation:

$$F_{rm} = k_r \cdot \left(6 + \frac{4n}{n_r}\right) \cdot \left(\frac{d_m}{100}\right)^2,$$

where F_{rm} – minimal radial load, кН; k_r – minimal load coefficient; n – rotating speed, rpm; n_r – nominal rotating speed, rpm; d_m – pitch bearing diameter, mm.

For the chosen bearing with coefficients k_r=0.15, n_r=14000 rpm minimal load at rotating speed of 20000 rpm accounts for 0.28 kN. At the same time there should be balance between forces acting on the bearing (weight, unbalance forces, inertia forces) and clearance in the bearing. When unbalanced force is insufficient and clearance is big, the shaft trunnion will perform chaotic motion in clearance, the bearing rolls will not be loaded, and the bearing will fail. Figure 10 shows motion of the rotor system at small unbalanced force. The rotor makes chaotic motion almost in the whole rotation range, where unbalanced force does not load the roll bearing and does not take it to running-in.

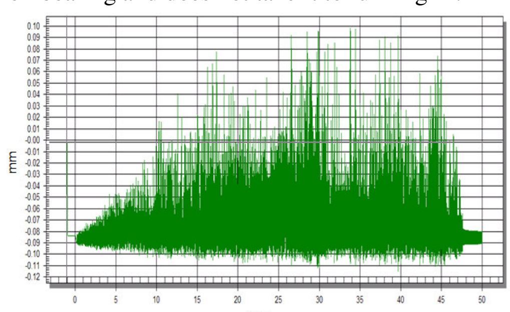
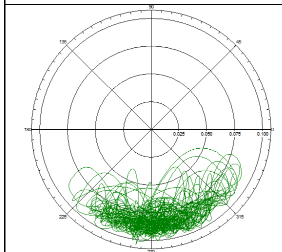
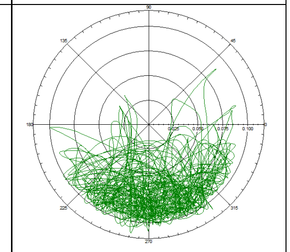


Figure 10 – Amplitude-time characteristic of rotor system
(clearance in bearing is equal to 0.08 mm, unbalance - 3 gcm)

This fact also follows from consideration of the rotor orbits motion (Table 4) and waterfall diagram of vibration spectra, Figure 11.

Table 4 – Rotor orbits (clearance in bearing is equal to 0.08 mm, unbalance - 3 gcm)

7.5 sec	30 sec
	
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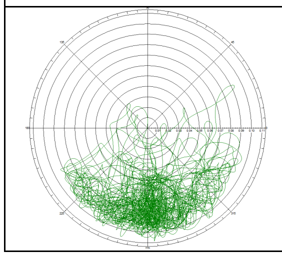	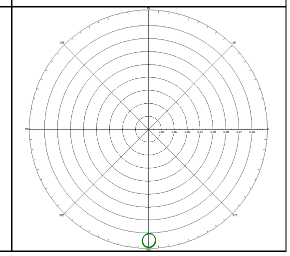

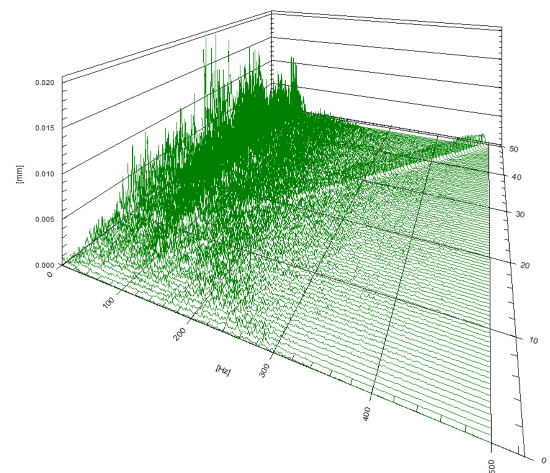

Figure 11 – Waterfall diagram of vibration spectra of rotor system
(clearance in bearing is equal to 0.08 mm, unbalance - 3 gcm)

Figure 12 shows unbalance influence on the rotor system behaviour.

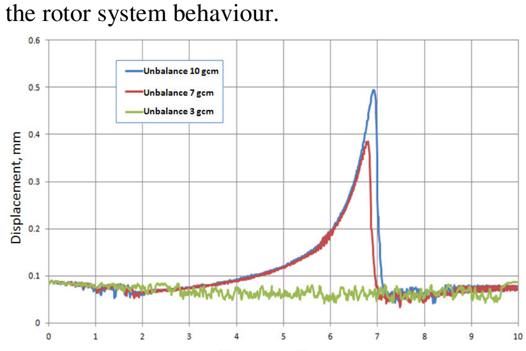

Figure 12 – Influence of rotor unbalance on

amplitude-frequency characteristic of rotor system (clearance in bearing is equal to 0.08 mm)

Comparing the forces acting on the bearing of the investigated rotor (clearance in bearing is equal to 0.08 mm) and obtained reactions in the bearing (Figure 13), it may be noticed that if unbalanced force does not exceed weight force in 2-3 times, then the rotor will make chaotic motion. Minimal load on the investigated bearing according to the SKF data will provide stable rotor operating without chaotic motion.

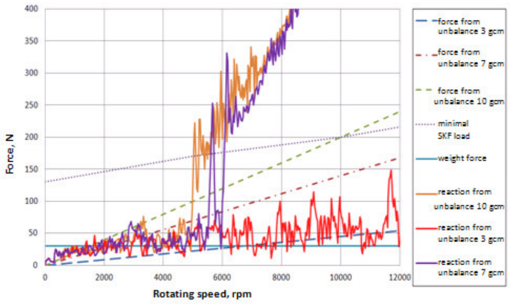

Figure 13 - Estimation of loads acting on bearing, bearing reactions and minimal load acting on bearing

Figure 14 presents clearances influence on the rotor system behaviour. With decrease of clearance in the bearing, the bearing work becomes stabler, inclination to chaotic motion decreases.

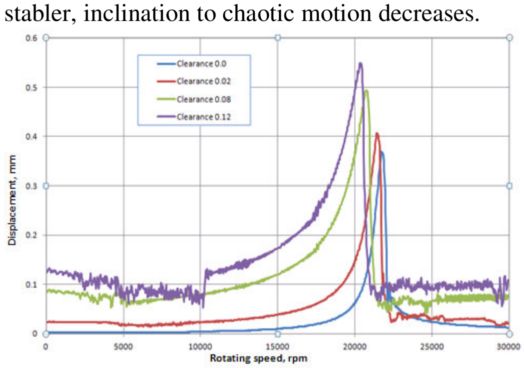

Figure 14 – Influence of roll bearing clearance on amplitude-frequency characteristic of rotor system (unbalance is equal to 10 gcm)

Conclusions

The present article may be considered as teaching material for students and entry-level engineers involved into the rotor systems design. The results of calculation and analysis of characteristics of the rotor system with the roll bearing are given. They show necessity of preliminary dynamic research of the rotor systems at nonlinear dynamic statement in order to choose the bearing type, its sizes and clearances, minimal rotor unbalance, i.e. parameters providing with stable and safe operating of the rotor system and finally the bearings durability.

It should also be noticed that investigation of the rotor systems supported by the angular-contact bearings is a separate task. However, some investigation results presented in the article may be extended to them.

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